

ARTNeT- KRI Capacity Building Workshop on Trade Policy Analysis:  
Evidence-based Policy Making and Gravity Modelling for Trade Analysis  
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## Session 3-4: Estimating the gravity models

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# Overview of the workshop

**Day 1 AM: Introduction to evidence-based policy making (EBPM) and tools**

**Day 1 PM: Introduction to gravity model for trade analysis**

- Its concepts, applications, weakness, and development

**Day 2 AM: Estimating the gravity models in STATA**

- Estimating intuitive gravity models
- Estimating trade potential using a gravity model
- Problems of intuitive gravity models

**Day 2 PM: Theoretical gravity models**

- Theoretical gravity models
- Econometric approaches to estimating theoretical gravity models

**Day 3: Advanced issues and consolidation**

- Recent development in gravity techniques
- Group exercises and presentation
- Workshop wrap-up

# Introduction

- The basic gravity model picks up some important empirical regularities, but has been posited without any explicit theoretical foundation.
- If we add in some micro-foundations, hopefully we will be able to derive something that looks a lot like gravity, but deals with the relative cost problem.

# Theoretical gravity models

# The theoretical gravity model

- A number of papers try to lay theoretical foundations to gravity model
- For the starting point, we will focus on Anderson & Van Wincoop (AvW), 2003. “The gravity with gravitas”.
- There are several theoretical gravity models developed for particular purposes.
  - Ex. Helpman et.al. (2008), Chaney (2008).
  - Showing complex empirical issues that OLS cannot handle
  - Sources such as the gravity course on Ben Shepherd’s website at <http://www.developing-trade.com/> provide rich details.

# AvW (2003)

- The most formal benchmark for theoretical gravity model so far
  - Bringing the gravity model a step closer to GE effects
  - Accounting for “relative price effects” on trade flow

*Things affecting “**relative price**” can influence bilateral trade flow. No matter the “things” happen between the two trading partners or happen with third parties.*

# Building gravity on micro-foundation

- Consumers have love of variety preferences with CES structure
- Producers are under monopolistic competition
- Introducing trade costs, and related domestic and foreign prices
- Closing the model with macroeconomic accounting identities

# AvW (2003) empirical model

$$\log X_{ij}^k = \log Y_i^k + \log E_j^k - \log Y^k + (1 - \sigma_k) [\log \tau_{ij}^k - \log \Pi_i^k - \log P_j^k]$$

$X_{ij}^k$  Sector-k exports from country i to j

$Y^k$  World income from sector k

$Y_i^k$  Exporting country's income from sector k

$E_j^k$  Importing country's expenditure on sector k

$\tau_{ij}^k$  Sector-k trade costs between country i and country j

# AvW (2003) empirical model

$$\log X_{ij}^k = \log Y_i^k + \log E_j^k - \log Y^k + (1 - \sigma_k) [\log \tau_{ij}^k - \log \Pi_i^k - \log P_j^k]$$

Outward multi-lateral resistance (MTR): looking from export sides

$$\Pi_i^k = \sum_{j=1}^C \left\{ \frac{\tau_{ij}^k}{P_j^k} \right\}^{1-\sigma_k} \frac{E_j^k}{Y^k}$$

Exports from country i to j depend on trade costs across ALL possible export markets.

Inward multi-lateral resistance (MTR): looking from import side

$$P_j^k = \sum_{i=1}^C \left\{ \frac{\tau_{ij}^k}{\Pi_i^k} \right\}^{1-\sigma_k} \frac{Y_i^k}{Y^k}$$

Imports of country j from country i depend on trade costs across ALL possible suppliers.

# AvW (2003) empirical implications

- The intuitive gravity model have **omitted variables**.
  - Serious consequences (bias and inconsistency)
- “**Relative trade costs**” matter, NOT absolute trade costs,
  - Two types of trade costs have to be taken into account
    - Trade costs between i and j
    - Trade costs of i and j with third parties
- For each observation, trade flows are “**unidirectional**” trade flows, NOT total trade of a country pair.
- Variables are in “**nominal**” terms, NOT real terms
  - (Special) price indices are captured separately in the MRT terms
- Using “**aggregate**” GDP, NOT GDP per capita

## AvW (2003) empirical implications

- Need the estimated “trade costs”, NOT just distance in gravity models.

- The base line can be

$$\log \tau_{ij}^k = b_1 \log \text{distance}_{ij} + b_2 \text{contig} + b_3 \text{comlang\_off} + b_4 \text{colony} + b_5 \text{comcol}$$

- Policy-related variables can be augmented into the trade cost functions.

- $\hat{b}$  is NOT purely trade cost elasticity, but combined with elasticity of substitution ( $\sigma_k$ )

# Estimating the AvW gravity model

$$\log X_{ij} = \log Y_i + \log Y_j - \log Y + (1 - \sigma)[\log \tau_{ij} - \log \Pi_i - \log P_j]$$

- The “**multilateral trade resistance (MTR)**” terms are empirically unobservable
- Two possible strategies to estimate the model
  - Fixed effects (FE) estimation
  - Baier and Bergstrand (2009) approach

# FE estimation

The theoretical gravity model:

$$\log X_{ij} = \log Y_i + \log Y_j - \log Y + (1 - \sigma)[\log \tau_{ij} - \log \Pi_i - \log P_j]$$

FE transformation of the model:

$$\log X_{ij} = C + F_i + F_j + (1 - \sigma)[\log \tau_{ij}]$$

where

$$C = -\log Y$$

$$F_i = \log Y_i - \log \Pi_i$$

Exporter fixed effects

$$F_j = \log Y_j - \log P_j$$

Importer fixed effects

$$\log \tau_{ij} = b_1 \log distance_{ij} + b_2 contig_{ij} + b_3 comlang\_of\ f_{ij} + b_4 colony_{ij} + b_5 comcol_{ij}$$

Estimated trade cost

# Estimation procedures

1. Generate (numerical) fixed effects dummies
  - Exporter & Importer dummies
  - Year dummies (if relevant)
  - Sector dummies (if relevant, but may be impractical)
2. Assuming key OLS assumptions are fine, we estimate the FE model with OLS
  - No multicollinearity
  - Homoskedasticity
  - $E$  is uncorrelated with any independent variables

# Estimation procedures

3. Variables that vary in the same dimensions as the FEs CANNOT be included in the model
  - Having multicollinearity problems with FEs
    - ex. MFN tariffs, ETCR scores
  - MUST be transformed into a new variable that vary bilaterally before including in the model,
  - or using alternative approaches.

# Estimating a FE gravity model with OLS in Stata

```
egen exporters=group(exp)  
egen importers=group(imp)
```

```
regress ln_trade ln_distance contig comlang_off colony comcol ///  
i.exporters i.importers, robust cluster(dist)
```

Alternatively,

```
quietly tabulate exporters, generate(exp_dum_)  
quietly tabulate importers, generate(imp_dum_)
```

```
regress ln_trade ln_distance contig comlang_off colony comcol ///  
exp_dum_* imp_dum_*, robust cluster(dist)
```

# Taking account of MTR

- How serious is this OV bias empirically?

quietly regress ln\_trade ln\_gdp\_exp ln\_gdp\_imp ln\_distance contig comlang\_off colony  
estimates store Basic

quietly regress ln\_trade ln\_distance contig comlang\_off colony comcol i.exporters i.importers  
estimates store FE

suest Basic FE, robust cluster(dist)

Number of obs = 1203

(Std. Err. adjusted for 803 clusters in distw)

	Coef.	Robust Std. Err.	z	P> z	[95% Conf. Interval]	
<b>Basic_mean</b>						
ln_gdp_exp	1.229322	.0334791	36.72	0.000	1.163704	1.29494
ln_gdp_imp	.8111193	.0270496	29.99	0.000	.758103	.8641356
ln_distance	-1.621796	.1310139	-12.38	0.000	-1.878579	-1.365013
contig	.393265	.2990856	1.31	0.189	-.192932	.9794619
comlang_off	1.402018	.2448563	5.73	0.000	.9221083	1.881928
colony	.4143014	.6492913	0.64	0.523	-.8582862	1.686889
comcol	.5482557	.2418329	2.27	0.023	.074272	1.022239
_cons	-28.74489	1.655698	-17.36	0.000	-31.99	-25.49978
<b>Basic_invar</b>						
_cons	1.792471	.05532	32.40	0.000	1.684046	1.900896
<b>FE_mean</b>						
ln_distance	-2.36174	.1027043	-23.00	0.000	-2.563037	-2.160444
contig	.2640105	.3103421	0.85	0.395	-.3442488	.8722699
comlang_off	.5179525	.2101043	2.47	0.014	.1061556	.9297493
colony	.2716033	.6763897	0.40	0.688	-1.054096	1.597303
comcol	.9207591	.20534	4.48	0.000	.5183	1.323218

# Taking account of MTR

- Distance coefficients
  - The intuitive model =  $-1.62^{***}$
  - The FE model =  $-2.36^{***}$
  - The difference is statistically significant at the 1% level

```
test [Basic_mean]ln_distance=[FE_mean]ln_distance

( 1)  [Basic_mean]ln_distance - [FE_mean]ln_distance = 0

           chi2( 1) =    35.55
       Prob > chi2 =    0.0000
```

# Augmented theoretical gravity models

- Policy variables often vary only in the exporter or importer dimensions
  - Perfectly collinear with corresponding FEs
- Possible Solutions:
  - Transforming the policy variables so as to vary by country pair (and continue using FE approach):
    - Sum of exporter and importer values.
    - Average of exporter and importer values, etc.
  - Using random effects (RE approach)
    - Ex. Egger (2002), Carrère (2006)
    - Not recommend due to a strong assumption.
  - Baier and Bergstrand (BB) approach
    - See Baier and Bergstrand (2009)

# An augmented FE model

```
gen tariff_sim_both = (tariff_exp_sim+tariff_imp_sim)/2
```

```
regress ln_trade tariff_sim_both ln_distance contig comlang_off colony  
comcol i.exporters i.importers, robust cluster(dist)
```

Linear regression

Number of obs = 858  
F( 73, 519) = 59.53  
Prob > F = 0.0000  
R-squared = 0.8324  
Root MSE = 1.6728

(Std. Err. adjusted for 520 clusters in distw)

ln_trade	Coef.	Robust Std. Err.	t	P> t	[95% Conf. Interval]	
tariff_sim_both	-.0115273	.0262997	-0.44	0.661	-.0631942	.0401396
ln_distance	-2.308299	.1139819	-20.25	0.000	-2.532222	-2.084377
contig	.1410702	.3245166	0.43	0.664	-.4964573	.7785977
comlang_off	.3168254	.2650492	1.20	0.232	-.2038758	.8375266
colony	.2959934	.8147513	0.36	0.717	-1.304622	1.896609
comcol	.9728162	.2414758	4.03	0.000	.4984261	1.447206

# Dummies and Fixed Effects in STATA

- Option 1: enter the dummies manually and use OLS:

```
egen exporters=group(exp)
```

```
egen importers=group(imp)
```

```
regress ln_trade tariff_sim_both ln_distance contig comlang_off colony comcol  
/// i.exporters i.importers, robust cluster(dist)
```

- Option 2: use a panel estimator (OLS + a trick) to account for one set of dummies:

```
xtset exporters
```

```
xtreg ln_trade tariff_sim_both ln_distance contig comlang_off colony comcol  
/// i.importers, robust fe
```

With the same clustering specification, results should be identical between regress with dummy variables and *xtreg, fe*

## Note:

- *xtreg* can only include fixed effects in one dimension. For additional dimensions, enter the dummies manually.
- For FE with more than 1 dimension:

*reg2hdfe* --estimation of a Linear Regression Model with two High Dimensional Fixed Effects

*ivreg2hdfe*-- estimation of a linear IV regression model with two high dimensional fixed effects.

# Sectoral gravity models

- Dummy variables need to be specified in the importer-sector, exporter-sector, and sector dimensions, because:

$$\ln(X_{ij}^k) = \ln(Y_i^k) + \ln(E_j^k) - \ln(Y^k) + (1 - \sigma_k) [\ln \tau_{ij}^k + \ln \Pi_i^k + \ln P_j^k]$$

- In addition, trade costs need to be interacted with sector dummies in order to take account of varying elasticities of substitution across sectors.

- Depending on the level of sectoral disaggregation used, this approach can result in huge numbers of parameters.
- Models can take a long time, and a big computer, to estimate.
- It is usually much easier to estimate separate models for each sector.

# Gravity (FE) models with panel data

- Panel data (cross-sectional time-series data) are data where multiple cases (people, firms, countries etc.) were observed at two or more time periods
- MTRs vary over time. So, ideally, the gold standard specification is using pair, importer-time and exporter-time fixed effects and identifies the effect of bilateral time-varying variables.

$$x_{ijt} = g(\pi_{it} + \chi_{jt} + \mu_{ij} + \eta_1 \tau_{ijt}) + \varepsilon_{ijt}$$

The model contains a huge number of dummies.

They are always perfectly collinear with policy variables.

# Disadvantages of the FE approach

- Losing insights on interested policy impacts
- Dimensionality constraints
  - A large number of dummies if running a sectoral gravity model: Dummies for sectors, exporter-sector, and importer-sector.
  - A larger number of dummies if they are time-variant
  - A trick: reduce dimensions by estimating separate models for each (major) dimension.

# RE approach

- Advantages:
  - No dimensionality constraints
  - Can include policy variables that collinear with FEs
- Disadvantages
  - A strong assumption: MTRs must be normally distributed across countries, with a given standard deviation.

(AvW model tells us that multilateral resistance is important, but it doesn't tell us anything about its distribution.)

- Otherwise, RE estimates are not consistent

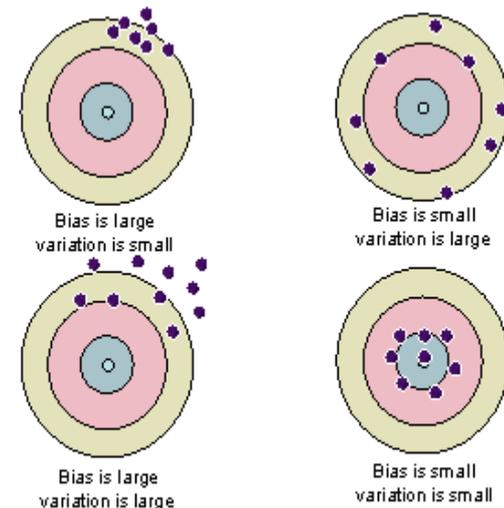
“FE estimates are always consistent even if the true model is RE.”

# Testing for FE vs. RE

- Fixed effects estimates are always **consistent**, even if the “true” model is random effects.
- Random effects are only **consistent** if the “true” model is random effects, in which case they are also **efficient**.
- Intuitively, random effects are an acceptable simplification when the difference between the two sets of coefficients is small.
- The Hausman test formalizes this intuition: it tests for a statistically significant difference between the two sets of coefficients. If the null hypothesis is rejected, random effects are inappropriate (inconsistent).
- If the null hypothesis is not rejected, random effects are an acceptable simplification, but either estimator can be used.

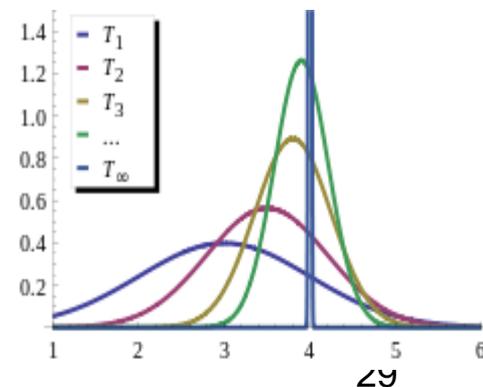
# Note: Quality of an estimator

- Unbiased - The expected value (mean) of the estimate's sampling distribution is equal to the underlying population parameter; that is, there is no upward or downward bias.
- Efficiency - While there are many unbiased estimators of the same parameter, the most efficient has a sampling distribution with the smallest variance.
- **Consistency** - Larger sample sizes tend to produce more accurate estimates; that is, the sample parameter converges on the population parameter.



Accuracy versus Quality of an Estimator Using Bias and Variation as Measurable Quantities Respectively

Consistent estimators are convergent and *asymptotically* unbiased (hence converge to the correct value): individual estimators in the sequence may be biased, but the overall sequence still consistent, if the bias converges to zero. Conversely, if the sequence does not converge to a value, then it is not consistent, regardless of whether the estimators in the sequence are biased or not.



# Testing for FE vs RE

- In Stata, adopt the following procedure to test for fixed versus random effects:
- Estimate the fixed effects model without the robust option, then issue the command `estimates store fixed`.
- Estimate the random effects model without the robust option, then issue the command `estimates store random`.
- Issue the command `hausman fixed random`.

quietly xtreg ln\_trade tariff\_sim\_both ln\_distance contig comlang\_off colony comcol i.importers, fe estimates store fixed

quietly xtreg ln\_trade tariff\_sim\_both ln\_distance contig comlang\_off colony comcol , re estimates store random

hausman fixed random

. hausman fixed random

	Coefficients		(b-B) Difference	sqrt(diag(V_b-V_B)) S.E.
	(b) fixed	(B) random		
tariff_sim~h	-.0115273	-.0633972	.0518699	.
ln_distance	-2.308299	-1.596022	-.7122775	.
contig	.1410702	1.156218	-1.015148	.
comlang_off	.3168254	.7041612	-.3873358	.
colony	.2959934	.0715757	.2244177	.
comcol	.9728162	.2792815	.6935347	.

b = consistent under Ho and Ha; obtained from xtreg

B = inconsistent under Ha, efficient under Ho; obtained from xtreg

Test: Ho: difference in coefficients not systematic

chi2(6) = (b-B)'[(V\_b-V\_B)^(-1)](b-B)

= -167.39      chi2<0 ==> model fitted on these data fails to meet the asymptotic assumptions of the Hausman test; see [suest](#) for a generalized test

**Ho: Estimates from FE and RE are not statistically different**

**Negative test statistics (invalid) !**

- To ensure that a Hausman test stat are positive value:

*hausman fixed random, sigmamore*

or

*hausman fixed random, sigmaless*

```
. hausman fixed random, sigmamore
```

	— Coefficients —		(b-B) Difference	sqrt(diag(V_b-V_B)) S.E.
	(b) fixed	(B) random		
tariff_sim~h	-.0115273	-.0633972	.0518699	.0353809
ln_distance	-2.308299	-1.596022	-.7122775	.0890378
contig	.1410702	1.156218	-1.015148	.1476931
comlang_off	.3168254	.7041612	-.3873358	.2590394
colony	.2959934	.0715757	.2244177	.2140335
comcol	.9728162	.2792815	.6935347	.1956301

b = consistent under Ho and Ha; obtained from xtreg

B = inconsistent under Ha, efficient under Ho; obtained from xtreg

Test: Ho: difference in coefficients not systematic

chi2(6) = (b-B)'[(V\_b-V\_B)^(-1)](b-B)  
 = 119.99  
 Prob>chi2 = 0.0000

Ho: Estimates from FE and RE are not statistically different

Ho is rejected.  
 RE estimates may be inconsistent.

Intuitively, if random effects estimates are very close to fixed effects estimates, then using random effects is probably an appropriate simplification. If the estimates are very different, then fixed effects should be used.

- Beware: the Hausman test has poor properties in practice!
- Stata's implementation has to use non-robust covariance matrices.
- Sometimes, the test statistic cannot even be calculated due to a breakdown in its assumptions.
- If you are serious about random effects, try the Hausman test, but take it with a grain of salt.

- *xtoverid* test (after *xtreg, re*) should always be preferred to the Hausman test because it allows for cluster-robust standard errors

```
quietly xtreg ln_trade tariff_sim_both ln_distance contig comlang_off colony ///  
comcol, re vce(cluster exporters)
```

*xtoverid*

```
Test of overidentifying restrictions: fixed vs random effects  
Cross-section time-series model: xtreg re robust cluster(exporters)  
Sargan-Hansen statistic 72.754 Chi-sq(6) P-value = 0.0000
```

# Baier and Bergstrand (BB) Approach

- Taking account of MTR without using dummies
- (Policy) variables that varies by exporter or importer can be directly included
- Doing a 1<sup>st</sup> order Taylor series approximation of the MTR terms.

# BB (2009) approach

Original form of Theoretical Gravity

$$\log X_{ij}^k = \log Y_i^k + \log E_j^k - \log Y^k + (1 - \sigma_k) [\log \tau_{ij}^k - \log \Pi_i^k - \log P_j^k]$$

Baier and Bergstrand transformation

$$\log X_{ij}^k = \log Y_i^k + \log E_j^k - \log Y^k + (1 - \sigma_k) [\log \tau_{ij}^{k*}]$$

by using the 1<sup>st</sup> order Taylor series approximation of MTR:

$$\log \tau_{ij}^{k*} = \log \tau_{ij}^k - \sum_{j=1}^N \theta_j^k \log \tau_{ij}^k - \sum_{i=1}^N \theta_i^k \log \tau_{ji}^k + \sum_{i=1}^N \sum_{j=1}^N \theta_i \theta_j \log \tau_{ij}^k$$

weighted by  
GDP shares

$$\theta_i^k = \frac{Y_i^k}{Y^k}$$

# BB estimation procedures

1. Calculate the weight terms
2. Calculate  $\log \tau_{ij}^*$  for EACH trade-cost variable

$$\ln dist_{ij}^* = \ln dist_{ij} - \sum_i \theta_i \ln dist_{ij} - \sum_j \theta_j \ln dist_{ij} + \sum_i \sum_j \theta_i \theta_j \ln dist_{ij}$$

$$contig_{ij}^* = contig_{ij} - \sum_i \theta_i contig_{ij} - \sum_j \theta_j contig_{ij} + \sum_i \sum_j \theta_i \theta_j contig_{ij}$$

3. Estimate the BB gravity model with OLS

$$\log X_{ij}^k = \log Y_i^k + \log E_j^k - \log Y^k + (1 - \sigma_k) [\log \tau_{ij}^{k*}]$$

# Points to keep in mind

- Need to apply the Taylor approximation to ALL variables in the trade cost function

$$\log \tau_{ij}^k = b_1 \log \text{distance}_{ij} + b_2 \text{contig} + b_3 \text{comlang\_off} + b_4 \text{colony} + b_5 \text{comcol}$$

- Possible endogeneity problems when using GDP weights
  - BB (2009) recommend using simple averages rather than the GDP-weighted averages

# BB with simple averages in Stata

Find the weight term:  $\theta_i^k = \frac{1}{N_i^k}$

```
sum exporters importers /**Results show that there are 58 exporters and importers**/
```

Variable	Obs	Mean	Std. Dev.	Min	Max
exporters	3598	29.46998	16.68045	1	58
importers	3598	29.46998	16.68045	1	58

Calculate  $\log \tau_{ij}^*$

```
egen temp1 = mean(ln_distance), by (exp sector)
```

```
egen temp2 = mean(ln_distance), by (imp sector)
```

```
egen temp3 = sum(ln_distance), by (sector)
```

```
gen ln_distance_star = ln_distance - temp1 - temp2 + (1/(58*58))*temp3
```

# BB with simple averages in Stata

Note: we assume, for brevity, distance is the only trade cost variable.

## A simplified BB model

```
. regress ln_trade ln_distance_star ln_gdp_exp ln_gdp_imp, robust cluster(dist)
```

Linear regression

```
Number of obs = 1203  
F( 3, 802) = 685.94  
Prob > F = 0.0000  
R-squared = 0.6492  
Root MSE = 2.4524
```

(Std. Err. adjusted for 803 clusters in distw)

ln_trade	Coef.	Robust Std. Err.	t	P> t	[95% Conf. Interval]	
ln_distance_star	-2.118955	.140292	-15.10	0.000	-2.394338	-1.843572
ln_gdp_exp	1.24344	.0316747	39.26	0.000	1.181265	1.305615
ln_gdp_imp	.7953212	.02684	29.63	0.000	.7426363	.8480062
_cons	-42.16343	1.210976	-34.82	0.000	-44.54048	-39.78637

## A simplified FE model

```
Number of obs = 1203
```

(Std. Err. adjusted for 803 clusters in distw)

	Coef.	Robust Std. Err.	z	P> z	[95% Conf. Interval]	
FE_mean ln_distance	-2.503828	.1119539	-22.36	0.000	-2.723254	-2.284403

Often-made mistakes

# Things to do and don't

1. **Do** aiming for theoretical gravity models, **Not** the intuitive ones
2. **Do** using “unidirectional” trade flows on LHS, **Not** the total trade of a country pair.
  - $X_{ij}$  or  $M_{ij}$  (NOT  $T_{ij}$ )
3. **Do** using nominal terms, **Not** real terms
  - Price indices are already captured by the MRT terms
4. **Do** using aggregate GDP, **Not** GDP per capita

# Things to do and don't

5. The “**estimated trade cost**” is needed, **Not** just distance.

ex.  $\log \tau_{ij}^k = b_1 \log \text{distance}_{ij} + b_2 \text{contig} + b_3 \text{comlang\_off} + b_4 \text{colony} + b_5 \text{comcol}$

6. Always think about **dimensions** of your dataset

- How many dimensions of your variables? exporter, importer, sector, time (and interactions among them)
- For sectoral gravity models, the simplest approach is to estimate separately, sector by sector.

7. Control variables that vary in the same dimensions as the FEs **CANNOT** be included in FE models.

- **Do** transform them if FE approach will be used (but it comes with a cost).
- BB approach can be an alternative.

# Things to do and don't

8. If using BB approach, **Do** apply the Taylor approximation to **ALL** trade cost variables.
9. Be **extremely** cautious when interpreting the gravity results with possible endogeneity problems
  - The estimated coefficients will be biased
  - Possible solutions are using panel data, instrumental variable techniques, etc.

# Reference

- Empirical examples uses the Uncomtrade dataset on bilateral trade in goods downloaded from WITS. The original trade data has been re-aggregated and compiled by to include GDP data are from the WDIs. Geographical data are from CEPII. Tariff data are from UNCTAD's TRAINS database downloaded from WITS).
- The completed lists of references are provided in the reading list

# Thank you!

Training materials of this session are drawn largely from Ben Shepherd (2012).

Available at

<http://www.unescap.org/tid/publication/tipub2645.asp>

