DYNAMICS IN HUMAN DEVELOPMENT: PARTIAL MOBILITY AND "JUMP"

*Atanu Sengupta and Abhijit Ghosh**

Development economists have argued that interesting dynamics exist in the intergroup movement of countries in terms of several development parameters. However, standard mobility measures are aggregative in nature. They do not study intergroup variation in mobility. In an earlier paper, we have introduced the concept of partial mobility for analysing the movement of a particular group. In this paper, the degree to which the group has progressed (or declined) from its current position is measured. It is argued that any movement is not sufficient to enhance (or worsen) a group's welfare. There is a perceived threshold, and any movement above that threshold may be defined as "jump". The focus of this paper is jumps, not just mobility. Jump is characterized with a set of axioms. The analysis of global data reveals that the poor countries fail to cross a threshold level, although there may be some limited movement within a narrow limit.

JEL Classification: O130, O400, O160, I310.

Key words: Human development, mobility indices, welfare implications.

I. INTRODUCTION

When an aggregative macro change is considered, it generally shields microscopic changes. For example, when considering the expansion of per capita gross domestic product (GDP), the movement of income of several subgroups is

* Atanu Sengupta, Associate Professor, Department of Economics, Burdwan University, Burdwan, West Bengal, India (Tel: +91-9832174249; e-mail: sengupta_atanu@yahoo.com); and Abhijit Ghosh, Assistant Professor of Economics, Division of Economics, A N Sinha Institute of Social Studies, Patna, Bihar, India (Tel: +91-9434334071; e-mail: abhijitghosh2007@rediffmail.com). Views expressed in the paper are those of the authors and not of the organizations to which they belong. The authors are indebted to Lawrence Katz and Raj Chetty for some helpful comments on an earlier draft of the paper. The authors also wish to thank the anonymous referee for useful comments.

generally relegated to the background (Quah, 1993; Baulch and Hoddinott, 2000; Basu, 2001). Similarly, consideration of the human development of a country generally blurs the view of how various subsections fare in terms of human entitlement (Grimm and others, 2010).

There are two ways in which this dilemma may be mitigated. One is the standard "inequality adjusted" growth rate and directional mobility indices[1] (Demuynck and Van de Gaer, 2011) and the other, the concept that is pursued here. The standard literature reveals some sort of additivity of individual positions (or decomposability of the aggregate into individual parts). In fact, the subgroup decomposability axiom basically declares this aspect in a precise mathematical way. The other is the "partial approach"[2] based on positional objectivity (Sen, 1993), which denies such decomposability or additivity, although accepts differences in opinion about changes that may not be additive or decomposable in a direct way. However, such differences in viewing dynamics would be reflected in the social choice decision rule.

The concern here is the dynamics of human development as indexed by mobility measures. However, mobility indices summarize the transition probabilities as represented by a transition matrix, but there are two deficiencies in using these indices. First, these are directionless and hence ethic neutral. They simply state the intensity (or probability) of movement without inferring whether the movement is for better or worse. Second, they are aggregate; there is no scope for observing the mobility of a particular group or subset of observations. In fact, these two perspectives are correlated. Aggregate ethical indices have very little significance except its connection with individual welfare (Sen, 1993). **Partial mobility indices** were developed by the authors to bring out these concerns (Sengupta and Ghosh, 2010).

It is possible to show that, even though a unique scalar mobility index that captures the so-called *concrete* reality – "the view from nowhere" (Sen, 1993) – may exist, agent-relative evaluations can be captured by partial mobility indices. Examples used there also showed that these agent-relative views might widely differ from each

[1] There is a long list of literature. For details, see Demuynck and Van de Gaer (2011).

[2] A very similar approach is the "spatial partial approach" that has been in use in the Public Facility Location models (Nijkamp, 2000).

other as well from that aggregative (or absolutist) view.[3] In the paper, positive and negative movement is identified from a different positional standpoint.

There are two objectives in the present paper. First, the axiomatic framework of Sengupta and Ghosh (2010) is generalized. Further, an attempt is made to develop a new concept of partial mobility, which is denoted as "**JUMP**". The concept of jump implies certain "lumpiness" in the partial mobility indices. The intention of the authors is simple: partial mobility of different cross-sectional units may be considered over a pre-specified time period. It may then be seen whether the mobility is substantial enough to warrant a change (increase or decrease) in welfare, which brings one to the notion of jump. For example, if two countries, "A" and "B", having literacy rates 20 per cent and 90 per cent, respectively, are considered for, say, a 20-year period, it is simply not enough to judge whether literacy has improved. The rate of change should also be known. For country B with the 90 per cent literacy rate, a small rise is acceptable. This is not the case for country A. A threshold level may be set for country A; it should jump (cross at least this threshold level) to produce sustainable[4] improvement in literacy. Any other change is illusive.[5]

There may be various reasons for this. Basically it is implied that there are certain ranges over which the partial mobility measures become insensitive to changes. This is not a problem of redefinition of classes – deciles, percentile or otherwise. It exists in the absolute valuation. It is possible to define classes that outstrip the problem. However, the problem remains. The question merely boils down to this: **How does one define classes so that the insensitivity is avoided?**

In this paper, an attempt is made to build an axiomatic framework incorporating such jumps. The method is then applied to a global data set to illustrate this difference between mobility and jump, and its implications for human welfare.

---

[3] In fact, a good example may be extracted from Dasgupta (2008), who considered the story of two girls: Becky in a suburban town in the American Midwest and Desta in rural Ethiopia. He argued that the lives of these two girls (who are "intrinsically very similar") are so distinct from each other (Desta is well embedded in household chores and hopes to be a housewife as is her mother, while Betty aspires to be a physician and hopes to live a life much richer than that of her parents) that they could be imagined to be living in different worlds. In conclusion, Dasgupta (2008) asserted, "In this article, I have used Becky's and Desta's experiences to show how it can be that the lives of essentially very similar people can become so different". What is important here is that these otherwise identical girls differ in one crucial aspect – they are in different positions and thus experiencing differential mobility rates. Much like the relativistic twins of Einstein, the time flows for one at a fast pace and for the other, at a snail's pace, if at all, giving rise to the "twins paradox".

[4] In a sense, no change is truly sustainable because there is always the possibility of falling back. One can consider the case of a sudden stock market crash. However, such possibilities are remote.

[5] One can use any other name such as "pace of poverty reduction". However, any such measure should be necessarily discrete. No continuous measure could serve this purpose.

The paper is divided into five sections. In section II, there is a very brief discussion of the new framework. In section III, the framework is extended to incorporate a new dimension – the deterioration in relative position. Section IV contains an empirical illustration using the state-level performance of India in the context of human development according to the authors' approach. Section V contains conclusions on the findings.

II. A BRIEF REVIEW OF THE PARTIAL MOBILITY AND JUMP

Conceptualizing partial mobility

The Human Development Index is constructed from a set of available data regarding the socioeconomic profile of a country. In order to evaluate positional mobility in human development, the first target is to transform these data into grouped data.[6] In any given time period t, it is possible to arrange these values (e_i^t) into intervals of equal length (d_i^t) starting from the lowest level (lowest value of e_i^t) to the highest ($e_i^t = e_i^{max}$). It is then possible to construct a transition probability matrix or mobility matrix between two time periods t and $t + \tau$ with $\tau > 0$. The transition probability is defined as:

$$p_{jj'} = \mathrm{Prob}\left(e_i^{t+\tau} \in d_{j'}^{t+\tau} \middle| e_i^t \in d_j^t\right) \quad (1)$$

where $\sum_{j'} p_{jj'} = 1$.

The transition probability ($p_{jj'}$) shows the probability of an observed unit moving from the jth class to the j$^{'th}$ class during the time span τ.

The relevant mobility matrix (P) (assuming that there are k intervals) may thus be written as:

[6] In Sengupta and Ghosh (2010), partial mobility was stated in a relativistic framework. In the present paper, also for empirical exercise, the authors have used that same framework. However, the partial approach developed does not depend on such a relativistic framework. It can be easily defined on the absolute space with the same set of axioms proposed. The emphasis is on **partiality** and not on relativity or positionality. It is however true that negative mobility is more often found in a relative framework. Absolute decline in human development is rare in normal circumstances, although it may appear in conditions of social or natural turbulence, such as war, famine and ethnic conflict. The idea of **jump**, on the other hand, is very relevant for the absolute case. Hence, it would be wise to keep the underlying framework open so that the researcher may adopt the partial or absolute approach according to his or her requirements.

$$P = \begin{pmatrix} p_{11}\, p_{12} \ldots\ldots p_{1k} \\ p_{21}\, p_{22} \ldots\ldots p_{2k} \\ \vdots \\ \vdots \\ p_{k1}\, p_{k2} \ldots\ldots p_{kk} \end{pmatrix} \quad (2)$$

The mobility index is now defined as:

$$M\ (P) = f\ (p_{ij}, i, j = 1, 2, \ldots\ldots k \mid p_{ij} \in P) \quad (3)$$

The mobility index can capture relative movements that are outside the purview of growth rate. However, there is no unique way of transforming a mobility matrix into the scalar index. A number of mobility indices are proposed in the literature (Shorrocks, 1976; 1978; Bhattacharya, 1995; Fields, 2001). Mobility is defined in different contexts (Fields, 2001). However, since this paper uses positional mobility, the axioms in matrix notation are retained.

The definition of mobility index provided in equation (3) spans the entire domain of P. This is surely not always desirable. The case discussed by Bhattacharya (1995) may be considered. He discussed mobility across different employment intervals or spells (proportion of time a person gets regular employment within a stipulated time, such as total hours within a working day, total days within a month or year). Instead of concentrating on all the different types of employment spells, from the human development perspective a subset of spells (preferably at the lower end) may be considered. This may produce a new type of mobility index that may be called a **subset mobility index**:

$$SM\ (P) = f\ (p_{ij}, i, j = 1, 2, \ldots\ldots k \mid p_{ij} \in P\ \&\ i'' \le i \le i') \quad (3a)$$

This measure is still not ethical. It is neutral as to whether one moves to a higher cell or to a lower cell. An **ethical upward mobility index** may be defined as:

$$EM\ (P) = f\ (p_{ij}, i, j = 1, 2, \ldots\ldots k \mid p_{ij} \in P\ \&\ i < j) \quad (3b)$$

This is a stricter version of $EM(P)$. A weaker version posits $i \le j$. A downward ethical mobility index can be similarly defined. While the definition (3) is clear, it is not so with definition (3a). In the aggregate ethical mobility index $EM(P)$, some sort of utility comparisons that may not be always tenable are inherently assumed.

A subset mobility measure involving the ethical dimension can be designated as **subset ethical mobility indices**, which is:

$$SUM(P) = f\,(p_{ij}, i, j = 1, 2, ... k \mid p_{ij} \in P \;\&\; i'' \le i \le i' \;\&\; i < j) \tag{3c}$$

This is a stricter version of *SUM(P)*. A weaker version posits $i \le j$. A downward subset mobility index can be similarly defined. Gang, Landon-Lane and Yun (2002) proposed a measure of upward and (downward) mobility that captures average conditional probability of moving to a higher class. However, the terms upper and lower must be properly interpreted. An upward movement does not automatically imply a rise in human welfare and vice-versa. For instance, in the example above, if an attempt had been made to find mobility across unemployment spells rather than employment spells, a movement to an upper cell would obviously imply a fall (rather than a rise) in human welfare.

Sengupta and Ghosh (2010) tried to move to a more disaggregate level. They introduced the concept of **partial mobility indices with directional element**. In their analysis, each interval (or class) represents a particular section. Since each row corresponds to an interval at the time period *t*, mobility with respect to a row marks the movement of that interval over the entire time point. They defined positive, negative and net partial mobility indices as follows:

$$M_+^i\,(P) = f\,(p_{i'j}, i', j = 1, 2, ... k \mid i = i' \;\&\; i < j) \tag{4a}$$

$$M_-^i\,(P) = f\,(p_{i'j}, i', j = 1, 2, ... k \mid i = i' \;\&\; i > j) \tag{4b}$$

$$NM^i\,(P) = M_i^+\,(P) - M_i^-\,(P) \tag{4c}$$

A positive partial mobility index captures improvement in position. Negative mobility, on the other hand, indicates a deterioration in position. Net mobility indices capture the net effect. Net mobility shows the difference between the probability of improvement and that of deterioration. For example, if a unit lies in a certain cell, there are possibilities for improvement and deterioration. However, what is the net change in welfare that can be expected? Net mobility provides an answer to this question.

However, in defining these new indices, the axiomatic structure is also reformulated. A set of axioms was postulated by Sengupta and Ghosh (2010) to define these partial indices (Sengupta and Ghosh, 2010). These are briefly discussed here. The normalization axiom (**NO**) (Shorrocks, 1978) constraining mobility indices between zero and unity are applicable for both positive and negative mobility. For net mobility, the absolute value is constrained.

Monotonicity (**MO**) is an important axiom. It relates the direction of change in the mobility index to the nature of *P*. In Shorrocks (1978), a matrix with very high number on the diagonals indicates low mobility. Conversely, a matrix that has the same numbers in every entry, given that the number must sum to 100 along each row, shows an extraordinarily high rate of mobility. However, these concepts are to be radically altered in the new framework. The concepts are generalized here symbolically as follows:

(a) (**MO**): If the off-diagonal entries of matrix P is greater than or equal to the corresponding off-diagonal elements of matrix P1, then M(P) ≥ M(P1) (Shorrocks, 1978);

(b) (**SMO**): If MO refers to a subset of rows (but not necessarily the entire matrix), this produces “subset monotonicity” (SMO);

(c) (**PMO**): If MO is applied to a single row, this produces “partial monotonicity” (PMO);

(d) (**MO+**): If the *right* off-diagonal entries of matrix P are greater than or equal to the corresponding off-diagonal elements of matrix P1, then M(P) ≥ M(P1);

(e) (**SMO+**): If MO+ is applied to a subset of rows (but not necessarily the entire matrix), this produces subset positive monotonicity (SMO+);

(f) (**PMO+**): If MO is applied to a single row, this produces partial positive monotonicity (PMO+) (Sengupta and Ghosh, 2010);

(g) (**MO-**): If the *left* off-diagonal entries of matrix P are less than or equal to the corresponding off-diagonal elements of matrix P1, then M(P) ≥ M(P1);

(h) (**SMO-**): If MO- is applied to a subset of rows (but not necessarily the entire matrix), this produces subset negative monotonicity (SMO-);

(i) (**PMO-**): If MO- is applied to a single row, this produces partial negative monotonicity (PMO-) (Sengupta and Ghosh, 2010).

Aggregate measures such as *EM(P)* satisfy MO+ and a similarly defined negative index will satisfy MO-. The subset measure *SM(P)* will satisfy SMO and positive subset measure *SUM(P)* satisfy SMO+.[7] Also, a *neutral* partial measure ($M^i(P)$) defined as:

$$M^i(P) = f\left(p_{i'j},\ i',\ j = 1, 2, \ldots\ldots k \,\middle|\, i = i'\right) \tag{3d}$$

7 A similarly defined negative subset mobility index will satisfy SMO-.

satisfies PMO.[8] It is clear $M_{+}^{i}(P)$ that satisfies PMO+ while $M_{-}^{i}(P)$ satisfies PMO-.

$$P^{ex} = \begin{pmatrix} 0.8 & 0.1 & 0.1 \\ 0.2 & 0.2 & 0.6 \\ 0 & 0.7 & 0.3 \end{pmatrix} \qquad P^{ex1} = \begin{pmatrix} 0.8 & 0.1 & 0.1 \\ 0.2 & 0.1 & 0.7 \\ 0 & 0.8 & 0.2 \end{pmatrix}$$

(a) By MO (Shorrocks, 1978), the first matrix has lower mobility;

(b) By MO+, the first matrix has lower mobility;

(c) By MO-, the first matrix has higher mobility;

(d) By PMO, the second and third row of the first matrix have lower mobility;

(e) By PMO+, the second row of the first matrix has lower mobility;

(f) By PMO-, the third row of the first matrix has higher mobility.

The case of perfect mobility (PM) and immobility (PI) is similar. For perfect mobility, the authors argue that:

(a) (**PM**): If P has identical rows, M(P) = 1 (Shorrocks, 1978);

(b) (**SPM**): If for a subset of rows (but not necessarily the entire matrix), the corresponding diagonal element is zero then the value of mobility index is one (SPM);

(c) (**PPM**): If for a single row, the corresponding diagonal element is zero then the value of mobility index is one (PPM);

(d) (**PM+**): For all rows, if the sum of the LEFT off-diagonal entries and the diagonal entry is zero, then M(P) = 1;

(e) (**SPM+**): If PM+ is applied to a subset of rows (but not necessarily the entire matrix), this produces subset positive PM (SPM+).

(f) (**PPM+**): If PM+ is applied to a single row, this produces partial positive PM (PPM+) (Sengupta and Ghosh, 2010);

(g) (**PM-**): For all rows, if the sum of the RIGHT off-diagonal entries and the diagonal entry is zero, then M(P) = 1;

(h) (**SPM-**): If PM- is applied to a subset of rows (but not necessarily the entire matrix), this produces subset negative PM (SPM-);

---

8 Many of these measures are for illustrative purpose only. The study is concentrated on ethical partial measures.

(i) (**PPM-**): If PM- is applied to a single row, this produces partial negative PM (PPM-) (Sengupta and Ghosh, 2010).

It is evident that that $M_{+}^{i}(P)$ satisfies PPM+ while $M_{-}^{i}(P)$ satisfies PPM-.

$$P^{ex2} = \begin{pmatrix} 0 & 0.1 & 0.9 \\ 0.9 & 0 & 0.1 \\ 0.1 & 0.9 & 0 \end{pmatrix}$$

(a) PPM implies that the second row is perfectly mobile;

(b) PPM+ implies that the first row is perfectly mobile;

(c) PPM- implies that the third row is perfectly mobile;

(d) However, for P^{ex2} there are no two identical rows.

Also in the case of perfect immobility, it can be argued that:

(a) (**PI**): If P = I, M(P) = 0 (Shorrocks,1978);

(b) (**SPI**): If for a subset of rows (but not necessarily the entire matrix), the corresponding diagonal element is one, the value of the mobility index is zero (SPI);

(c) (**PPI**): If for a single row the corresponding diagonal element is one, the value of the mobility index is zero (PPI).

However, for further development, a less stringent axiomization is required:

(a) (**PI+**): For all rows, if the sum of the RIGHT off-diagonal entries is zero, then M(P) = 0;

(b) (**SPI+**): If PI+ is applied to a subset of rows (but not necessarily the entire matrix), it produces subset positive PI (PPI+);

(c) (**PPI+**): If PI+ is applied to a single row, it produces partial positive PI (PPI+) (Sengupta and Ghosh, 2010);

(d) (**PI-**): For all rows, if the sum of the LEFT off-diagonal entries is zero, then M(P) = 0;

(e) (**SPI-**): If PI- is applied to a subset of rows (but not necessarily the entire matrix), it produces subset negative PI (SPI-);

(f) (**PPI-**): If PI- is applied to a single row, it produces partial negative PI (PPI-) (Sengupta and Ghosh, 2010).

It is evident that that $M_{+}^{i}(P)$ satisfies PPM+ while $M_{-}^{i}(P)$ satisfies PPM-.

$$P^{ex3} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

(a) PPI implies that the first row is perfectly immobile;

(b) PPI- implies that the second row is perfectly immobile;

(c) PPI+ implies that the third row is perfectly immobile;

(d) However, $P^{ex3} \neq I$.

Positive partial mobility and jump

A positive (negative) movement of smaller magnitude would obviously be less attractive than a movement of large magnitude. Moreover, the movement over some classes may yield no change in welfare – this insensitive range is defined as JUMP. There are at least two reasons for this. First, the changes have to be *perceptible* to have a meaningful impact. Second, the question of *sustainability*[9] arises – the changes must be sufficiently large to enable the unit to maintain its new position or at least reduce the risk of reverting back to the original position (poverty traps (Azariadis and Stachurski, 2005; Dasgupta, 2009) and vulnerability threshold).

In short, to have a meaningful discussion about the dynamics of human development, concentration should be directed towards analysing partial mobility above a certain minimum threshold level. This requires a redefinition of partial mobility in terms of some "discrete" changes. A movement with some specified magnitude is defined as a **jump.** However, the magnitude of this threshold is bound to be imprecise. The fact that human interaction is often based on such impreciseness is well emphasized by Basu (1994).

The authors first start with an aggregate jumping index. They propose as an example that a social thinker visualizes that a unit should move at least by an amount δ. The movement by this amount is then defined as a jump. All movements below this level would be irrelevant for that thinker.

[9] No change is truly sustainable because there is always the possibility of falling back; for example, the case of a sudden stock market crash may be considered. However, such possibilities are remote.

An (**aggregate**) **jumping index**[10] of magnitude (size) δ is then defined as:

$$J\,(P) = f\,(p_{ij},\, i, j = 1, 2, \ldots\ldots k \,\big|\, p_{ij} \in P \;\&\, \big|i - j\big| \geq \delta) \tag{5}$$

This aggregate index satisfies all the axioms of (aggregate) mobility (as defined by Shorrocks, 1978) except **MO**.

However, the measure is not adequate to capture partial changes. For that purpose, a new axiomatic structure and a mathematical characterization are needed. (The axiomatic structure is given in the technical appendix I.)

Now a set of (positive) partial jumping measures $J_+^i\,(P)$ is suggested as follows:

$$J_+^i\,(P) = \sum_{j=i^l+1}^{k} (p_{i^l j})^{\alpha} \tag{6}$$

where $\alpha \geq 1$ and $i^l = i + \delta$.[11]

These indices also have a strong intuitive interpretation if $\alpha = 1$. They give the probability of moving (by an amount of at least δ) to a higher category from the present category by the i^{th} group. They give a different partial view of the jumping. For other values of α, a linear transformation of this probability is obtained.

III. NEGATIVE MOVEMENT IN PARTIAL MOBILITY

Negative jumping indices

So far, only the issue of jumping or no-jumping towards a better position has been considered. This may be called an **optimistic view**. However, there is another possibility that was nascent in the example – the question of deterioration or moving down to a lower cell. Thus, the fact that movement from a higher cell to a lower cell could result in a fall in welfare is introduced, and it has been labelled “negative jumping”. This may be regarded as a **pessimistic view**.

[10] Any name may be used, such as “pace of poverty reduction”. However, any such measure should be necessarily discrete. No continuous measure could serve this purpose.

[11] This definitely assumes that δ is an integer. If not, then the *nearest* integer corresponding to $(i+\delta)$ must be taken; also, $\delta \geq 0$.

As before, there will be (k-1) partial negative mobility indices $M_{-}^{i}(P)$ with i = 2, ... k (in the stronger version). Each index summarizes the mobility from the point of view of the i^{th} class. $M_{-}^{2}(P)$ is defined as the **Rawlsian Ethical Mobility Index** (**REMI**), viewing mobility from the next most deprived category. Similarly, $M_{-}^{k}(P)$ can be defined as the **Elitist Ethical (negative) Mobility Index** (**EMI**), viewing mobility from the next to the best-endowed category.[12] Similarly, negative mobility can be redefined from the weaker viewpoint with i = 1, 2, ... k mobility indices.

A set of (negative)[13] partial jumping measures $J_{-}^{i}(P)$ is suggested as follows:

$$J_{-}^{i}(P) = \sum_{j=1}^{i'-1} (p_{i'j})^{\alpha} \tag{7}$$

where $\alpha \geq 1$ and $i' = i - \delta$.[14]

These indices also have a strong intuitive interpretation if $\alpha = 1$. They give the probability of moving (by an amount of at least δ) to a higher category from the present category by the i^{th} group. They give a different partial view of the jumping. For other values of α, a linear transformation of this probability is obtained.

These indices satisfy **NJMO**, **NENJ** and **NEPJ**[15] if α =1. However, the result does not carry over to the generalized jumping index with α >1.

Net jumping

Thus, to this point both the optimistic and the pessimistic views of jumping have been discussed. It is possible to interpret the positive jumping indices $J_{+}^{i}(P)$ as capturing *pull factors* – the factors that pull up a group to a better position. On the contrary, the negative mobility indices $J_{-}^{i}(P)$ summarize *push factors* – the factors that push down a group to a worse position. The interest is in assessing the net effect – whether it is the pull or the push factors that are stronger. However, in order to grasp net jumping, the positive and negative threshold should be of *comparable* magnitude to the group for which there was an attempt to unravel its jumping. The actual magnitude may differ in absolute terms but the group should *value* them equivalently.

---

[12] In certain cases, the lowest feasible category may not exist at time point t. In this case, one may move on to the least observable category. Similarly, an argument may be extended for the highest feasible category k.

[13] The adjective positive is cleared now.

[14] The argument is similar to that found at http://hdr.undp.org/en/reports/.

[15] See technical appendix II.

There will be k partial net mobility indices $NM^i(P)$ with $i = 2, ... k$. Each index summarizes the mobility from the point of view of the i^{th} class. $NM^1(P)$ is defined as the **Rawlsian**[16] **Ethical Mobility Index** (**REMI**), viewing mobility from the most deprived category. Similarly, $NM^k(P)$ can be defined as the **Elitist Ethical (negative) Mobility Index** (**EMI**), viewing mobility from the next to the best-endowed category.[17] For each partial mobility index there may be a number of jumping indices ($NJ^i(P)$). For the lowest group, **REJ** has already been defined. **Negative Rawlsian Extreme Jump** (**NREJ**) can be measured as the probability to stay in the lowest category. The difference between the two produces the **Net Ralwsian Extreme Jump** (**N-REJ**).

In considering the appropriate axiomatic structure, it is found that the net index will satisfy **M-NO** (Sengupta and Ghosh, 2010). The immediate implication of this axiom is simple: $-1 \leq NJ^i(P) \leq 1$. In essence then, there are three extreme values of $NJ^i(P)$: (a) the perfect no-jump, $NM^i(P) = 0$; (b) the perfect positive jump, $NJ^i(P) = 1$; and (c) the perfect negative jump, $NM^i(P) = -1$.

Thus, the twin aspects of mobility are well captured within a single measure. The same should apply for monotonicity (**MO**). The appropriate monotonicity axiom for the net measure should capture both these aspects. It is defined below.

(J.2)$^/$ **Modified Jumping Monotonicity** (**M-JMO**): **M-JMO** implies $J(P) > J(P^{**})$ if at least one of the following relations is true ($\left|i - j\right| > \delta$):

(a) $p_{ij} \geq p_{ij}^{**}$ for all $i < j$ and $p_{ij} = p_{ij}^{*}$ for some $i \neq j$;

(b) $p_{ij} \leq p_{ij}^{**}$ for all $i \geq j$ and $p_{ij} = p_{ij}^{*}$ for some $i \neq j$.

Thus, the partial mobility index satisfying **M-MO** turns out to be:[18] $j^i(P) = f\,(p_{ij}, i, j = 1, 2, ... k \left| i < j \right| \& \left| i - j \right| > \delta) - g\,(p_{ij}, i, j, = 1, 2, ... k \left| i \geq j \right| \& \left| i - j \right| > \delta)$.

Now it is posible to define net partial jumping indices[19] $NM^i(P)$ as follows:

$$NJ^i(P) = J_{+}^{i}(P) - J_{-}^{i}(P). \quad (8)$$

---

[16] The term is Rawslian is from Rawls (1971). Basu (2001) first used this term to define the lowest income category (bottom quintile) in his analysis.

[17] In certain cases, the lowest feasible category may not exist at time point *t*. In this case, one may move on to the least observable category. Similarly, the argument may be extended for the highest feasible category *k*.

[18] This specific form follows from the form of **M-MO**.

[19] Perfect jump or no-jump cannot be explicitly stated here. Since it is a net measure, the effect depends on the relative strength of the positive and negative components of the measure.

This measure satisfies both **M-NO** and **M-MO**.

For this paper, the authors have considered only net REMI (strong positive and weak negative version) and Ralwsian Extreme Jumps (positive, negative and net). These ethical mobility and jumping indices have been used to study the experience of India with human development in recent decades.

IV. GLOBAL EXPERIENCE OF PARTIAL AND FRACTIONAL MOBILITY

An important contribution in studying cross-country relative movement using the mobility matrix was made by Quah (1993). He used the Summer Heston data set to generate a mobility matrix for studying income mobility of a cross-section of countries over the period 1962-1984. He constructed cells in proportion to the global average income. His table (as reproduced in Ray, 1999, p. 19) reveals a very complex picture of income mobility that no simple growth rate (or mobility index) could capture.

In using an aggregative perspective, Ray (1999, p. 20) argued that “A matrix with very high numbers on the main diagonal ... indicates low mobility”. In a sense then, Quah’s mobility matrix represents a low level of mobility. However, what is the implication of this low mobility for a country’s welfare? The question remains unanswered in the aggregative context. The structure presented in this paper would help to sort out the problem.

The authors now focus on reinterpreting Quah’s mobility matrix from their perspective. For example, in their terminology, about 76 per cent of the world’s poorest (Rawlsian) countries in 1984 were consistent laggards. They remained at the Rawlsian (lowest) cell (one fourth of the global average income) both in 1962 and 1984. On the contrary, 95 per cent of the world’s richest (elitist) countries in 1984 were also the richest in 1962. No Ralwsian country was able to be moved to the “above-average” cell in any of those 22 years. In a relative sense, there has been little welfare improvement for the Rawlsian countries.

In term of jump, the picture is more succinct. Of the 24 per cent of the Rawlsian countries in 1984 that could be moved out of their cell, only 12 per cent could be moved to the level of half the global average, and the remaining 12 per cent could barely equal the global average. In the contrary, 5 per cent of the 1962 elites moved only one step below – that was still more than twice the world average. There was not one elitist country whose income fell below the global average in this time period. Thus, the elitist countries have definitely seen an improvement in welfare.

The measures developed above are readily applicable to the global data on human development. Such data are available from the website of United Nations Development Programme (UNDP).[20] The authors have selected a time span of 10 years covering three time points: 1997, 2002 and 2007.[21] These points were selected only to capture the era of globalization and integration of the world economy.

Three dimensions of human development have been selected: the Education Index (EI), Gross Domestic Product (GDP) Index and Life Expectancy (LE) Index together with the Human Development Index (HDI). There is some controversy regarding the interpretation of HDI. However, it has been widely accepted as an aggregative indicator of human achievement. As for the year-specific benchmark, it is incorporated within the *Human Development Report 2011: Sustainability and Equity – A Better Future for All*. Ranking might be a contentious issue, but only for countries that are close to each other in terms of ranking. For countries lying well apart from each other, the issue looses much of its significance.

The authors' analysis is given in the table below.[22] Relative mobility data are contained in tables A.1 to A.4 (see annex). In the original UNDP arrangement (*Human Development Report 2009*) the countries were divided into four categories: Very High Human Development; High Human Development; Medium Human Development; and Low Human Development. The same classification is maintained in the categorization of the mobility matrix. This would help to remove some of the ambiguity regarding the discrete classification in the mobility matrix.

There are numerous debates regarding the long-run performance in the global scenario. However, as the analysis is partial, it does not give any unilateral picture. In considering partial mobility, improvement in the partial mobility of the Rawlsian class may be clearly seen in the two separate time points regarding all the parameters of human development. However, net mobility, although decreasing, is still negative for those countries. As for the elitist category, the picture shows the opposite. While in respect of the category education there is some deterioration, there is clear improvement for the GDP index and all other parameters. In short, the benefits of globalization are reaching all the Rawlsian (poorest) countries at a very slow rate.

---

[20] http://hdr.undp.org/en/reports/.

[21] Three *Human Development Reports* for 1999, 2004 and 2009, respectively, cover the time periods mentioned.

[22] The methodology of calculation of partial mobility is given in Sengupta and Ghosh (2010).

Table 1. Partial mobility and jump mobility for human development indicators

HDI (Rawlsian)	Partial mobility		
	Strictly positive	Weakly negative	Net
1997-2002	0.15 (Imm.)	0.85 (Imm.)	-0.70
2002-2007	0.39 (Imm.)	0.61 (Imm.)	-0.32
1997-2007	0.41 (Imm.)	0.59 (Imm.)	0.02
HDI (Elitist)	**Weakly positive**	**Strictly negative**	**Net**
1997-2002	1	0	1
2002-2007	1	0	1
1997-2007	1	0	1

EI (Rawlsian)	Partial mobility		
	Strictly positive	Weakly negative	Net
1997-2002	0.19 (Imm.)	0.81 (Imm.)	-0.62
2002-2007	0.43 (Imm.)	0.57 (Imm.)	-0.14
1997-2007	0.52 (Imm.)	0.48 (Imm.)	0.04
EI (Elitist)	**Weakly positive**	**Strictly negative**	**Net**
1997-2002	1	0	1
2002-2007	0.94	0.06 (Imm.)	0.88
1997-2007	0.97	0.03 (Imm.)	0.94

LE (Rawlsian)	Partial mobility		
	Strictly positive	Weakly negative	Net
1997-2002	0.08 (Imm.)	0.92 (Imm.)	-0.04
2002-2007	0.32 (Imm.)	0.68 (Imm.)	-0.34
1997-2007	0.37 (Imm.)	0.63 (Imm.)	-0.26
LE (Elitist)	**Weakly positive**	**Strictly negative**	**Net**
1997-2002	1	0	1
2002-2007	1	0	1
1997-2007	1	0	1

Table 1. *(continued)*

GDPI (Rawlsian)	Partial mobility		
	Strictly positive	Weakly negative	Net
1997-2002	0.21 (Imm.) (**REJ** = 0.02)	0.79 (Imm.)	-0.58
2002-2007	0.25 (Imm.)	0.75 (Imm.)	-0.50
1997-2007	0.33 (Imm.) (**REJ** = 0.02)	0.67 (Imm.)	-0.26
GDPI (Elitist)	**Weakly positive**	**Strictly negative**	**Net**
1997-2002	1	0	1
2002-2007	1	0	1
1997-2007	1	0	1

Source: Authors' calculation.

Note: "Imm." implies movement to the immediate class; "REJ" Rawlsian Extreme Jump (described in the text).

The analysis of jump reveals a more succinct picture. It may be observed that, for the Rawlsian group (except in the case of GDPI), the probability of moving beyond the immediate class is zero. Thus, although there might be some mobility for the Rawlsian class, it is very weak. Very few of the Rawlsian countries could be moved from the "very poor" to "rich" or even the "moderate" category.

Interestingly, almost all of the Rawlsian countries are in Africa (see annex table A.5). Most of these countries remain at the Rawlsian level throughout the time span. On the contrary, all the elitist countries are industrialized developed countries, including some non-European countries, such as Australia and Japan. In short, even after the phenomenal expansion of the global economy, polarization is still prominent.

V. CONCLUSION

In this paper, an attempt was made to bring in partial changes with some quantitative threshold. A new concept of jump has been developed for this purpose. The analysis is then extended to global data covering country-specific data. It reveals that, even after the phenomenal expansion of the global economy, polarization is still prominent.

The inequality of global development would by itself not be a problem provided that it remains within tolerable limits and is not associated with extreme deprivation and poverty at the lower end. Unfortunately, the latter situation is the case. With such intolerable inequality, improvement in the position of the elitist countries alone cannot improve global welfare. Some countries require jumps to cross the barrier in order to become sustainable. However, the standard aggregative analysis cannot bring out this lopsided picture. Use of the techniques of partial mobility and jump has helped to unravel this scenario.

The position urgently needs some policy intervention. There should be greater integration in the global arena and targeted policy towards the Rawlsian countries in terms of aid (in cash and in kind) in order to tie them over during their current difficulties. The targeted help is required because in the past there were innumerable cases of leakage and failed programmes. It would be tragic if the same scenario were to be repeated. Democratization and transparency in programme formulation and implementation are required. Peer pressure from people and civil society is necessary. All this may now seem to be as old sayings go, but there is obviously no other path. We may take hope from the words of Indian Nobel laureate poet Rabindranath Tagore in this regard, “As I look around I see the crumbling ruins of a proud civilization strewn like a vast heap of futility. And yet I shall not commit the grievous sin of losing faith in Man. I would rather look forward to the opening of a new chapter in his history after the cataclysm is over and the atmosphere rendered clean with the spirit of service and sacrifice”-‘Crisis of Civilization’ – the last public speech which Tagore gave on the occasion of his eightieth birthday in April 1941. Provided that sufficient and judicious incentives are given, there is no reason why change cannot be achieved. Recent experience with random choice experiments supports such changes at the micro level. It is the accumulation of these small changes that brings about macro changes.

REFERENCES

Azariadis, C., and J. Stachurski (2005). Poverty traps. Article prepared for *The Handbook of Economic Growth,* P. Aghion and S. Durlauf, eds. Mimeo.

Basu, K. (1994). The traveller's dilemma: paradoxes of rationality in game theory. *American Economic Review*, vol. 84, No. 2, pp. 391-395.

________ (2001). On the goals of development. In *Frontiers of Development Economics: The Future in Perspective*, G. Meier and J.E. Stiglitz, eds. New York: Oxford University Press.

Baulch, B., and J. Hoddinott (2000). Economic mobility and poverty dynamics in developing countries. *Journal of Development Studies*, vol. 36, No. 6, pp. 1-24.

Bhattacharya, K. (1995). Employment fluctuations in rural India: a statistical analysis. PhD dissertation. Indian Statistical Institute, Kolkata, India.

Dasgupta, P. (2008). Mathematical economics and economic reasoning. In *Princeton Companion to Mathematics,* W.T. Growers, ed. Princeton, New Jersey: Princeton University Press.

________ (2009). Poverty traps: exploring the complexity of causation. In *Poorest and the Hungry: Assessments, Analysis and Action,* J. von Braun, R. Vargus Hill and R. Pandya-Lorch, eds. Washington, D.C.: International Food Policy Research Institute.

Demuynck, Thomas, and Dirk Van de Gaer (2011). Inequality adjusted income growth. Belgium: Ghent University and CORE Universite Catholique de Louvain. Mimeo. Later published in *Economica*, vol. 79, No. 316, pp. 747-765.

Fields, G.S. (2001). *Distribution and Development: A New Look at the Developing World*. New York: Russell Sage Foundation; and Cambridge and London: MIT Press.

Gang, I.N., J. Landon-Lane, and M. Yun (2002). Gender differences in German upward income mobility. *Journal of Applied Social Science Studies*, vol. 123, No. 1, pp. 3-14.

Grimm, M., and others (2010). Inequality in human development: an empirical assessment of 32 countries. *Social Indicators Research*, vol. 97, pp. 191-211.

Nijkamp, P., ed. (2000). *Handbook of Regional and Urban Economics: Regional Economics,* vol. 1. Amsterdam: North-Holland Publishing Co.

Quah, D. (1993). Empirical cross-section dynamics in economic growth. *European Economic Review*, vol. 37, pp. 426-434.

Rawls, J. (1971). *A Theory of Justice*. Oxford: Oxford University Press.

Ray, D. (1999). *Development Economics.* New Delhi: Oxford University Press India.

Sen, A. (1993). Positional objectivity. *Philosophy and Public Affairs*, vol. 22, pp. 126-145.

Sengupta, A., and A. Ghosh (2010). Negative and positive partial mobility: a study of the relative changes in human development. *Social Indicators Research*, vol. 99, No. 2, pp. 249-268.

Shorrocks, A.F. (1976). Income mobility and Markov assumption. *Economic Journal*, vol. 86, pp. 566-578.

________ (1978). The measurement of mobility. *Econometrica*, vol. 46, pp. 1013-1024.

United Nations Development Programme (various years). *Human Development Reports (various years)*. New York: Oxford University Press.

ANNEX

Table A.1. Intercountry relative Human Development Index dynamics, 1997, 2002 and 2007, first-order transition matrix, time stationary

(a) 1997-2002	Upper end point			
Number of countries	0.50	0.80	0.90	1.00
34	0.85	0.15	0.00	0.00
90	0.04	0.84	0.11	0.00
27	0.00	0.04	0.70	0.26
18	0.00	0.00	0.00	1.00
(b) 2002-2007				
Number of countries	0.50	0.80	0.90	1.00
33	0.61	0.39	0.00	0.00
82	0.00	0.72	0.28	0.00
29	0.00	0.00	0.62	0.38
25	0.00	0.00	0.00	1.00
(c) 1997-2007				
Number of countries	0.50	0.80	0.90	1.00
34	0.59	0.41	0.00	0.00
90	0.00	0.64	0.36	0.00
27	0.00	0.00	0.33	0.67
18	0.00	0.00	0.00	1.00

Source: Authors' calculation.

Table A.2. Intercountry relative Education Index dynamics, 1997, 2002 and 2007, first-order transition matrix, time stationary

(a) 1997-2002	Upper end point			
Number of countries	0.50	0.80	0.90	1.00
27	0.81	0.19	0.00	0.00
54	0.02	0.76	0.22	0.00
49	0.00	0.0816	0.7347	0.1837
39	0.00	0.00	0.00	1.00
(b) 2002-2007				
Number of countries	0.50	0.80	0.90	1.00
23	0.57	0.43	0.00	0.00
50	0.00	0.76	0.22	0.02
48	0.00	0.00	0.79	0.21
48	0.00	0.00	0.06	0.94
(c) 1997-2007				
Number of countries	0.50	0.80	0.90	1.00
27	0.48	0.52	0.00	0.00
54	0.00	0.63	0.35	0.02
49	0.00	0.00	0.65	0.35
39	0.00	0.00	0.03	0.97

Source: Authors' calculation.

Table A.3. Intercountry relative Life Expectancy Index dynamics, 1997, 2002 and 2007 first-order transition matrix, time stationary

(a) 1997-2002	Upper end point			
Number of countries	0.50	0.80	0.90	1.00
38	0.92	0.08	0.00	0.00
88	0.03	0.82	0.15	0.00
42	0.00	0.2007	0.86	0.2007
1	0.00	0.00	0.00	1.00
(b) 2002-2007				
Number of countries	0.50	0.80	0.90	1.00
38	0.68	0.32	0.00	0.00
78	0.00	0.78	0.22	0.00
49	0.00	0.10	0.43	0.47
4	0.00	0.00	0.00	1.00
(c) 1997-2007				
Number of countries	0.50	0.80	0.90	1.00
38	0.63	0.37	0.00	0.00
88	0.023	0.705	0.261	0.011
42	0.000	0.048	0.357	0.595
1	0.00	0.00	0.00	1.00

Source: Authors' calculation.

Table A.4. Intercountry relative Gross Domestic Production Index dynamics, 1997, 2002 and 2007, first-order transition matrix, time stationary

(a) 1997-2002	Upper end point			
Number of countries	0.50	0.80	0.90	1.00
52	0.79	0.19	0.00	0.02
81	0.04	0.86	0.10	0.00
25	0.00	0.04	0.48	0.48
11	0.00	0.00	0.18	0.82
(b) 2002-2007				
Number of countries	0.50	0.80	0.90	1.00
44	0.75	0.25	0.00	0.00
81	0.05	0.72	0.22	0.01
22	0.00	0.00	0.23	0.77
22	0.00	0.00	0.00	1.00
(c) 1997-2007				
Number of countries	0.50	0.80	0.90	1.00
52	0.67	0.31	0.00	0.02
81	0.025	0.654	0.259	0.062
25	0.00	0.00	0.08	0.92
11	0.00	0.00	0.00	1.00

Source: Authors' calculation.

Table A.5. Consistent Rawlsian and "elitist" countries over three time periods

Rawlsian countries	Elitist countries
Benin, Burkina Faso, Burundi, Central African Republic, Chad, Democratic Republic of the Congo, Côte d'Ivoire, Eritrea, Ethiopia, Guinea, Guinea-Bissau, Malawi, Mali, Mozambique, Niger, Sierra Leone, Rwanda, Senegal, Togo, Zambia	Australia, Austria, Belgium, Canada, Denmark, Finland, France, Germany, Iceland, Japan, Luxembourg, Netherlands, New Zealand, Norway, Sweden, Switzerland, United Kingdom, United States

Source: Authors' calculation.

TECHNICAL APPENDIX I

The (**Aggregate**) **Jumping Index**[a] of magnitude (size) δ is defined as:

$$J(P) = f(p_{ij}, i, j = 1, 2, ... k \mid p_{ij} \in P \;\&\; |i - j| \geq \delta) \quad (1)$$

This aggregate index satisfies all the axioms of (aggregate) mobility (as defined by Shorrocks, 1978) except **MO**. This is simply because the index *J(P)* is completely insensitive to all changes below magnitude δ. Thus, **MO** can be redefined as **AJMO** (**Jumping Monotonicity**):

(J.1) **Aggregate Jumping Monotonicity** (**AJMO**): For jumping, two mobility matrices, P and P**, are defined so that $p_{ij} \geq p_{ij}^{**}$ for all $i \neq j$ and $p_{ij} = p_{ij}^{**}$ for some $i \neq j$ where $|i - j| \geq \delta$, then **JMO** implies that JM(P) > JM(P**).

AJMO indicates that a rise in at least one of the non-diagonal p_{ij} for $|i - j| \geq \delta$, with others remaining the same, should raise *J.* In fact, **AJMO** is a truncated version of **MO.** The following proposition is then obvious:

Proposition 1: A jumping index *J* that satisfies **MO** will necessarily satisfy **AJMO** but the opposite is not true.

The frailties of this index are many. It completely ignores a group's aspirations and feelings. A jump of dimension δ may be quite substantial for one group[b] while it may not be worthwhile for another. The aggregate jump of δ is then meaningless. However, in this paper there is more concern with partial mobility since this appears to be more relevant for a meaningful analysis of human welfare.[c] Following Sengupta and Ghosh (2010), it is possible to construct a set of partial jumping indices that captures the jump for a particular group. The partial jumping measures may be defined as:

$$J^{i}(P) = f(p_{ij}, i, j = 1, 2, ... k \mid p_{ij} \in P \;\&\; i = i' \;\&\; i < j \;\&\; |j - i| \geq \delta_i) \quad (2)$$

(In the subsequent analysis, the suffix in the quantity δ_i will be suppressed for the sake of brevity in the analysis.)

---

[a] This could be called by another name, such as "pace of poverty reduction"; however, any such measure should be necessarily discrete. No continuous measure could serve this purpose.

[b] The argument here may be closely approximated by the concept of marginal utility.

[c] Such arguments are given in detail in Sengupta and Ghosh (2010).

In defining these jumping indices, it must be stressed that, unlike the aggregate jumping measure here, not only the degree but also the direction of change is important. Hence, it is necessary to bring out some ethical dimensions in jumping.

Although the concept of jumping is used for positional movement, it could be used in the space of absolute values. For example, the symmetric income mobility indices defined by Fields (2001) may be considered. In essence, they depend on the absolute difference of each observation between two time points ($\sum_i \left| x_t^i - x_{t+r}^i \right|$). This may be regarded as a *total income mobility index*. A jumping index may thus be defined as: $\sum_{i, \left| x_t^i - x_{t+r}^i \right| \geq \delta_i} \left| x_t^i - x_{t+r}^i \right|$. *Partial income jumping indices* may define the jump as visualized by *i*, which may depend on the absolute difference from the i^{th} observation- $\left| x_t^i - x_{t+r}^i \right|$ and only if -$\left| x_t^i - x_{t+r}^i \right| \geq \delta_i$. It is possible also to move on to a group jumping income index in this case. An individual in any society identifies himself or herself with a particular income group. That individual is concerned about jumping within that group. Thus, *group partial jumping* is $\sum_{(i), \left| x_t^{(i)} - x_{t+r}^{(i)} \right| \geq \delta_i} \left| x_t^{(i)} - x_{t+r}^{(i)} \right|$ where $x^{(i)} \in x^G$ and $x^G = \{x^{(1)}, x^{(2)}, ..., x^{(g)}\}$. Also $x = \{x^1, x^2, ..., x^n\}$ is the population and $x^G \subseteq x$.

In the alternative framework the jumping is emphasized from the view point of a particular section of the society. In the present analysis each interval (or class) represents a particular section. Since each row corresponds to an interval at the time period *t*, mobility with respect to a row shows the movement of that interval over the entire time point. There will thus be (*k*-1) partial mobility indices,[d] $M^i(P)$ with i = 1, 2, ... k – 1. Each index summarizes the mobility from the point of view of the i^{th} class. $M^i(P)$ can be defined to be the **Rawlsian Ethical Mobility Index** (**REMI**), viewing mobility from the most deprived category. Similarly, $M^{k-1}(P)$ can be defined to be the **Elitist Ethical Mobility Index** (**EMI**), viewing mobility from the next to the best-endowed category.[e] For each of these partial mobility indices there will be a corresponding number of jumping indices. A particular type of jump is the **extreme**

[d] Discussed here are *positive mobility indices* only, that is, a natural extension of Shorrock's measures. Subsequently, there is a discussion of *negative mobility*.

[e] In certain cases, the lowest feasible category may not exist at time point *t.* In such a case, it is possible to move on to the least observable category. Similarly, the argument may be extended for the highest feasible category *k*.

jump. An extreme jump for any i^{th} class is the jump from the class i to the class k. Of particular interest is the **Ralwsian Extreme Jump** (**REJ**) – a jump from the Ralwsian position to the elitist position.

It is obvious that the jumping index will satisfy **NO**. Since jumping is more disaggregated than the partial mobility index, the monotonocity axiom has to be redefined as:

(J.2) **Ethical Jumping Monotonocity** (**EJMO**): **EJMO** will imply M(P) > M(P*) if any of the two axioms are true: (a) $p_{ij} \geq p^*_{ij}$ for all $i < j$; and (b) $p_{ij} = p^*_{ij}$ for some $i < j$ with $\left|i - j\right| \geq \delta$ in both the cases. It can be seen that **EJMO** is a subset to the axiom **JMO.**

From this, it is possible to proceed to the construction of a set of partial jumping indices. The partial measures may be defined as: $M^i(P) = f\,(p_{i'j} \big| i = i', \& \left|i - j\right| \geq \delta)$. **NO** guarantees that the value of $M^i(P)$ should lie between zero and one. A necessary condition for satisfying **EJMO** is that the measure must become completely insensitive to all $p_{ij}\ \forall i \geq j$. In fact, **EJMO** defines a positive partial measure. Hence, the positive partial jumping measures is redefined as: $j^i_+(P) = f\,(p_{i'j} \big| i = i' \;\&\; i < j \;\&\; \left|i - j\right| \geq \delta)$ such that $0 \leq j^i_+(P) \leq 1$.

Now, the two extreme cases are considered: (a) when $J^i_+(P) = 0$ (perfect non-jump); and (b) when $J^i_+(P) = 1$ (perfect jump).

As for perfect no jump, the original axioms (Shorrocks, 1978; Sengupta and Ghosh, 2010) are still applicable. However, they are too strong a requirement in the case of partial jumping. A weaker version is applicable here. This may be stated as follows:

(J.3) **Ethical No Jump (ENJ):** $J^i(P) = 0$ when the probability of staying at any of the interval within i to $(i+\delta)^{th}$ position is unity. In other words, $J^i(P) = 0$ when $\sum_{j=i}^{i+\delta} p_{ij} = 1$.

If, in any index, **J** satisfies **EI** (Sengupta and Ghosh, 2010)**,** it will always satisfy **ENJ**. However, **ENJ** may be true[f] even if **EI** is not true. Moreover, the milder form of **EI,** namely **EPI** (Sengupta and Ghosh, 2010), is still valid for jumping.

[f] See the example below.

Again it can be seen that, if $\sum_{j=i}^{i+\delta} p_{ij} = 1$, **EPI** is valid. However, **EPI** is true[g] even when $\sum_{j=i}^{i+\delta} p_{ij} = 1$. Nonetheless, the discreteness of jumping breaks the nice logical relationship between No-Jump and Perfect-Jump. The next axiom defines **EPJ.**

(J.4) **Ethical Perfect Jump (EPJ)**: $J^i(P) = 1$ when there is zero probability for any observations (belonging to the i^{th} row) to move to any lower cell. **EPJ** ensures that $J^i\,(p_{i'j}, i', j = 1, 2, ...) = 0 \forall j \le i' \;\&\; \left|i' - j\right| > \delta$.[h]

The authors now bring out an example to demonstrate the relationship between these axioms. The following mobility matrix may be considered.

$$\mathbf{P^{ex1}} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0.5 & 0.5 \\ 0 & 0 & 0.5 & 0.5 \\ 0 & 1 & 0 & 0 \end{pmatrix}$$

Since $\mathbf{P^{ex1}} \neq \mathbf{I}$, Shorrock's axiom (**I**) is not applicable here. Hence, $M(\mathbf{P^{ex1}}) \neq 0$.[i] Also, since there are no identical rows, **PM** is not valid; thus, $M(\mathbf{P^{ex1}}) \neq 1$.[j] The three classes may now be considered. For the first group, $M^1(\mathbf{P^{ex1}}) = 0$ by **EI**, and by **ENJ** too, $J^1(\mathbf{P^{ex1}}) = 0$ no matter what is the value of δ_1; now for the second group, $M^2(\mathbf{P^{ex1}}) = 1$ by **EPM** (Sengupta and Ghosh, 2010). Again, **ENJ** implies that $J^2(\mathbf{P^{ex1}}) = 0.5$, if $\delta_2 = 1$, but is unity if $\delta_2 = 2$. For the third group, obviously **EI** is not applicable here, but **EPI** indicates that $M^3(\mathbf{P^{ex1}}) = 1$; however, assuming that $\delta_3 = 1$, by **ENJ**, $J^3(\mathbf{P^{ex1}}) = 0$. Again for the fourth group, both mobility and jumping (irrespective of the value of δ) is zero. Thus, all the different types of mobility and jumping experiences are available from the same transition matrix that does not match with the overall mobility of the transition matrix.

---

[g] Ibid.

[h] This certainly means that EPJ seems to imply total indifference to what happens to the various probabilities of upward jump. The introduction of such a preference pattern will necessitate the concept of weighted partial jumping indices. However, the authors have refrained from such an exercise in this paper.

[i] Strictly speaking this is a consequence of a strong immobility axiom.

[j] Again this is a consequence of a stronger version of the axiom.

A set of (positive) partial jumping measures $J_{+}^{i}(P)$ is suggested as follows:

$$J_{+}^{i}(P) = \sum_{j=i'+1}^{k} (p_{i'j})^{\alpha} \tag{3}$$

where $\alpha \geq 1$ and $i' = i + \delta$.[k]

These indices also have a strong intuitive interpretation if $\alpha = 1$. They give the probability of moving (by an amount of at least δ) to a higher category from the present category by the i^{th} group. They give a different partial view of the jumping. For other values of α, a linear transformation of this probability is obtained.

[k] This definitely assumes that δ is an integer. If it is not, then it is necessary to take the *nearest* integer corresponding to $(i+\delta)$; also, $\delta \geq 0$.

TECHNICAL APPENDIX II

Considered next is the relevant axiomatic structure that can define negative jumping. The normalization axiom (**NO**) will be still valid here. However, the negative monotonicity axiom (**NMO**) (Sengupta and Ghosh, 2010) can be weakened here. This is because the truncation involved in negative jumping. The **NMO** may be stated as:

(J.1)$^{/}$ **Negative Jumping Monotonocity axiom** (**NJMO**): **NJMO** will imply $J^i(P) > J^i(P^{**-})$ if the following relations are true: (a) $p_{ij} \leq p_{ij}^{**-}$ for all $i > j$ and (b) $p_{ij} = p_{ij}^{**-}$ for some $i > j$ and $\left|i - j\right| \geq \delta$.

However, unlike **EJMO, NJMO** will fail to be a subset of **AJMO**.

This is evident by simply expanding **AJMO. AJMO** will necessarily imply $P \succ P^{**}$ is true if either one of the two alternatives are true (for $\left|i - j\right| \geq \delta$):

(a) $p_{ij} \geq p_{ij}^{**}$ for all $i > j$ and $i < j$ together with $p_{ij} = p_{ij}^{**}$ for some $i > j$;

(b) $p_{ij} \geq p_{ij}^{**}$ for all $i > j$ and $i < j$ together with $p_{ij} = p_{ij}^{**}$ for some $i < j$.

Now if the second alternative is true then this necessarily implies that **EJMO** is satisfied. However, **NJMO** is not. In fact it contradicts **NJMO**.

In considering non-jumping, it may be noted that the axiom **EPNJ** has to be turned upside down in order to accommodate negative mobility indices.

(J.2)$^{/}$ **Negative Ethical Partial No-Jump** (**NEPNJ**): $J^i(P) = 0$ when the probability staying at any of the interval within i to $(i\text{-}\delta)^{th}$ position is unity. In other words, $J^i(P) = 0$ when $\sum_{i=j-\delta}^{i} p_{ij} = 1$ with $\delta \geq 0$.

Similarly, the Negative Ethical Perfect Jump axiom (**NEPJ)** can be reformulated.

(J.3)$^{/}$ **Negative Ethical Perfect Jump** (**NEPJ**): $J^i(P) = 1$ when there is zero probability for any observations (belonging to the i^{th} row) to move to any lower cell. **EPJ** ensures that for a unit valued positive partial jump for the position (row) $i = i'$: $p_{i^{/}j} = 0$ *if* $j \leq i^{/}$ & $\left|i - j^{/}\right| > \delta$.[l]

[l] This certainly means that **EPJ** seems to imply total indifference to what happens to the various probabilities of upward jump. Introduction of such a preference pattern will necessitate the concept of weighted partial jumping indices. The authors refrained from such an exercise in this paper.

As with positive jump, the nice symmetry between No-Jump and Perfect Jump is broken here.

In the second example, $M^4(\mathbf{P^{ex1}}) = -0.5$ by **EPNM**. However, $J^4(\mathbf{P^{ex1}}) = -1$ only if $\delta = 1$. For $\delta > 1$, however $J^4(\mathbf{P^{ex1}}) = 0$ by **EPNNJ**.

A set of (negative)[m] partial jumping measures $J_-^i(P)$ is suggested as follows:

$$J_-^i(P) = \sum_{j=1}^{i'-1} (p_{i'j})^\alpha \qquad (4)$$

where $\alpha \geq 1$ and $i' = i - \delta$.[n]

These indices also have a strong intuitive interpretation if $\alpha = 1$. They give the probability of moving (by an amount of at least δ) to a higher category from the present category by the i^{th} group. They give different partial view of the jumping. For other values of α, a linear transformation of this probability is obtained.

These indices satisfy **NJMO**, **NENJ** and **NEPJ** if $\alpha = 1$. However, the result does not carry over to the generalized jumping index (3) with $\alpha > 1$.

[m] The adjective positive is cleared now.

[n] The argument is similar to that available from http://hdr.undp.org/en/reports/.