Multiregional Input-Output Tables and Global Value Chain Analysis

Bangkok, Thailand

10 July 2019

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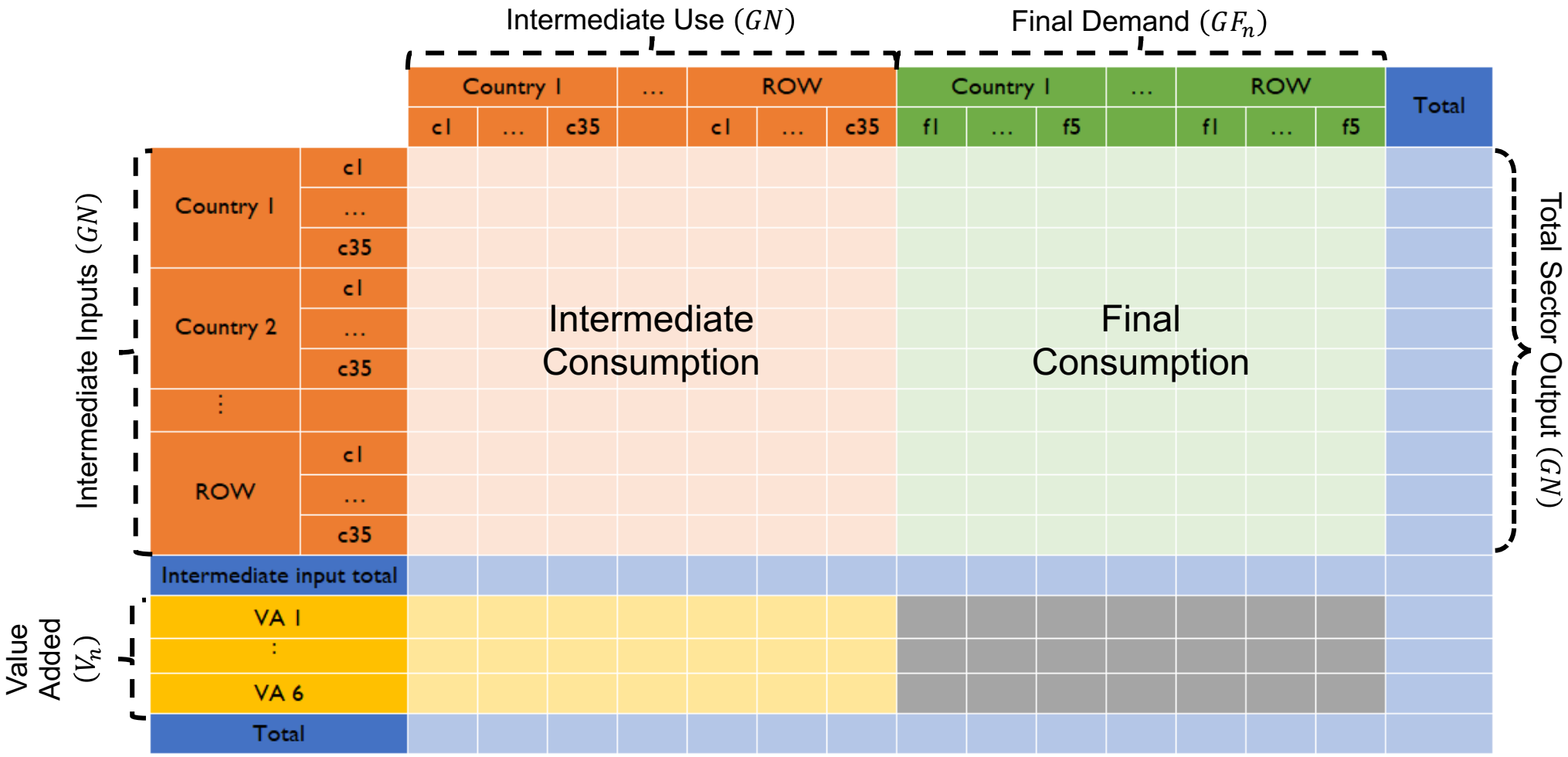
Outline

- MRIO
- Leontief Insight (1936)
- Indicators
  - The $\hat{V}B\hat{Y}$ Matrix
  - Decomposed Value-added Terms (Koopman, Wang, and Wei, *2014*; Wang, Wei, and Zhu, *2018*)
  - Revealed Comparative Advantage (Koopman, Wang, and Wei, *2014;* Wang, Wei, and Zhu, *2018*)
  - Participation in Global Value Chains (Wang, Wei, Yu, and Zhu, *2018*)
  - Upstreamness (Antràs et al., *2012*)

MRIO

Structure of the MRIO



Notation:

G = no. of countries including RoW

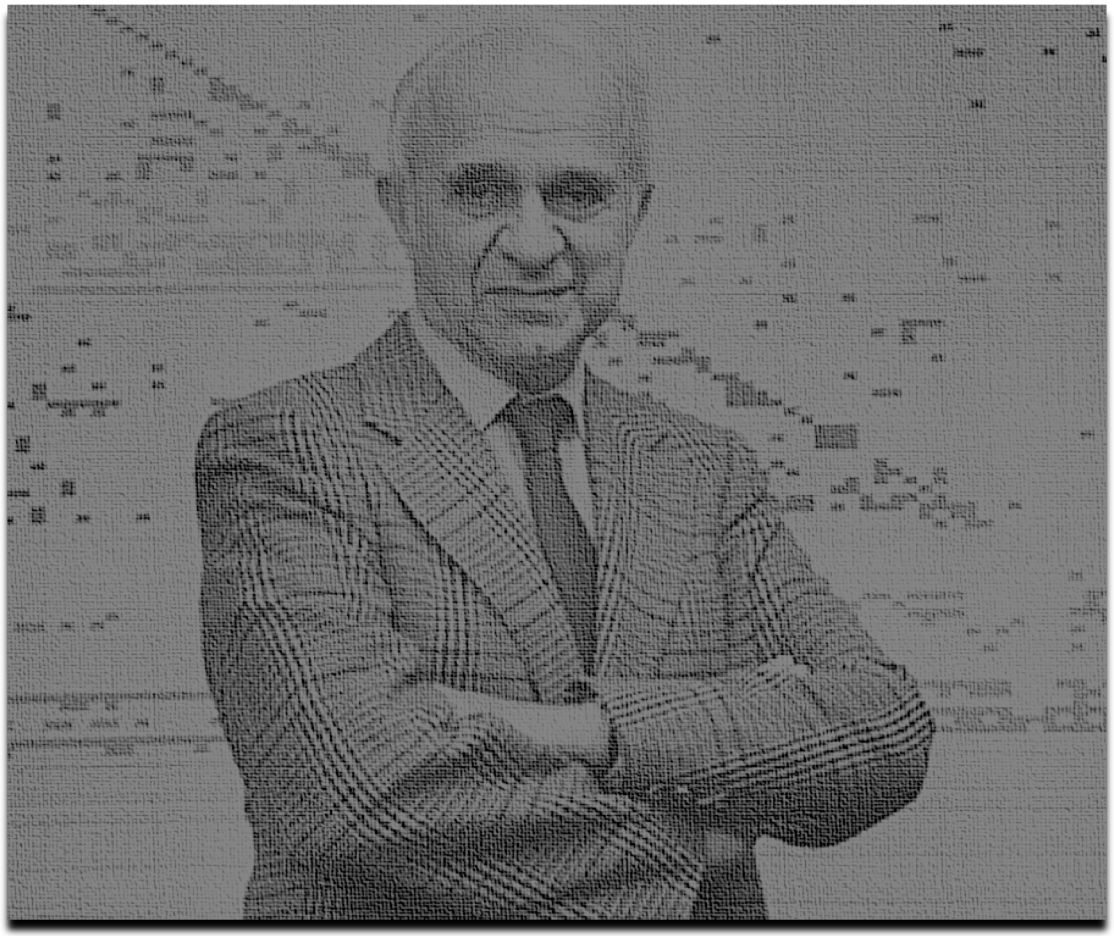
N = no. of sectors

F_n = no. of final demand components

V_n = number of value added components

The Leontief Insight (1936)

The Leontief Insight (1936)



Two-Country Production Case

$$X^s = A^{ss}X^s + Y^{ss} + A^{sr}X^r + Y^{sr} \quad r, s = 1,2$$

$$\begin{bmatrix} X^s \\ X^r \end{bmatrix} = \begin{bmatrix} A^{ss} & A^{sr} \\ A^{rs} & A^{rr} \end{bmatrix} \begin{bmatrix} X^s \\ X^r \end{bmatrix} + \begin{bmatrix} Y^{ss} + Y^{sr} \\ Y^{rs} + Y^{rr} \end{bmatrix}$$

Rearranging gives us the standard Leontief insight:

$$\begin{bmatrix} X^s \\ X^r \end{bmatrix} = \begin{bmatrix} 1 - A^{ss} & -A^{sr} \\ -A^{rs} & 1 - A^{rr} \end{bmatrix}^{-1} \begin{bmatrix} Y^{ss} + Y^{sr} \\ Y^{rs} + Y^{rr} \end{bmatrix} = \underbrace{\begin{bmatrix} B^{ss} & B^{sr} \\ B^{rs} & B^{rr} \end{bmatrix}}_{\text{Leontief inverse}} \begin{bmatrix} Y^s \\ Y^r \end{bmatrix}$$

The $\hat{V}B\hat{Y}$ Matrix

Two-Country Production Case

- The total value added induced by the $1 gross output is equal to the sum of direct and all rounds of indirect value added generated from the $1 of the gross output production process.

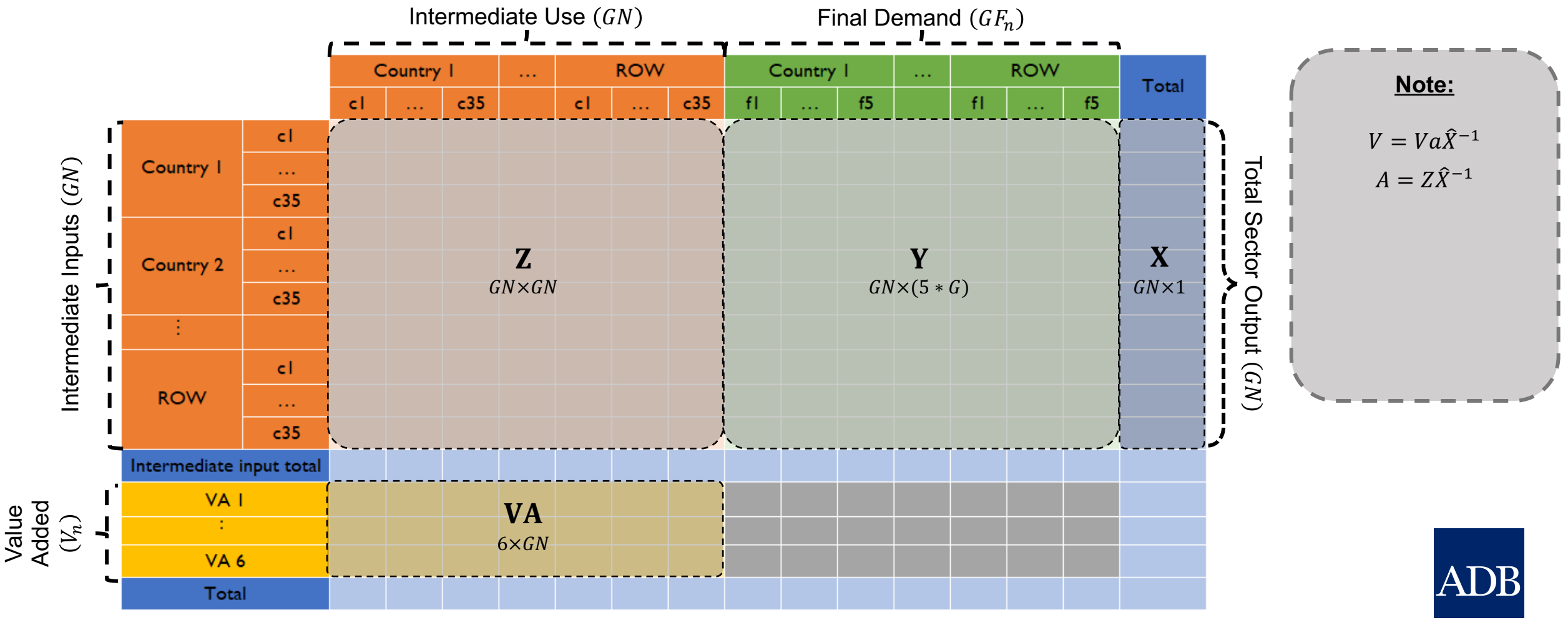
$$TVA = V + VA + VAA + VAAA + \cdots$$
$$= V(I + A + A^2 + A^3 + \cdots) = V(I - A)^{-1} = VB$$

Two-Country Production Case

- Decomposition of the country-sector level value added and final products production as a direct application of the standard Leontief decomposition:

$$\hat{V}B\hat{Y} = \begin{bmatrix} v_1^s b_{11}^{ss} y_1^s & v_1^s b_{12}^{ss} y_2^s & v_1^s b_{11}^{sr} y_1^r & v_1^s b_{12}^{sr} y_2^r \\ v_2^s b_{21}^{ss} y_1^s & v_2^s b_{22}^{ss} y_2^s & v_2^s b_{21}^{sr} y_1^r & v_2^s b_{22}^{sr} y_2^r \\ v_1^r b_{11}^{rs} y_1^s & v_1^r b_{12}^{rs} y_2^s & v_1^r b_{11}^{rr} y_1^r & v_1^r b_{12}^{rr} y_2^r \\ v_2^r b_{21}^{rs} y_1^s & v_2^r b_{22}^{rs} y_2^s & v_2^r b_{21}^{rr} y_1^r & v_2^r b_{22}^{rr} y_2^r \end{bmatrix}$$

Partitioning the MRIO



Note:

$$V = Va\hat{X}^{-1}$$

$$A = Z\hat{X}^{-1}$$

Interpreting the $\hat{V}B\hat{Y}$ Matrix: Summary

Row-wise Interpretation

- The sum of the $\hat{V}B\hat{Y}$ matrix along a row accounts for how each country's domestic value added originating in a particular sector is used by the sector itself and all its downstream countries-sectors.
- It traces **forward industrial linkages** across all downstream countries-sectors from a **producer's perspective**.
- Decomposes how each country's GDP by industry is used, directly or indirectly to satisfy domestic and foreign final demand.
- Decomposes a country-sector's GDP according to where it is used.
- **Who is the final consumer of value-added generated from a country-sector?**
- Johnson and Noguera (2012), Johnson (2014)

Column-wise Interpretation

- The sum of the $\hat{V}B\hat{Y}$ matrix down a column accounts for all upstream countries-sectors' value added contributions to a specific country-sector's final products.
- It traces **backward industrial linkages** across upstream countries-industries at different stages of production from a **user's perspective**.
- Decomposes a country-sector's final goods and services production into its original country-sector sources.
- Decomposes a country-sector's final production into all upstream country-sector's contributions to value added.
- **What is structure of the value-added source of a country-industry's final production?**
- Timmer et al. (2013, 2014)

Note: GDP = Final Demand in the aggregate, but not in the sector and bilateral-sector level.

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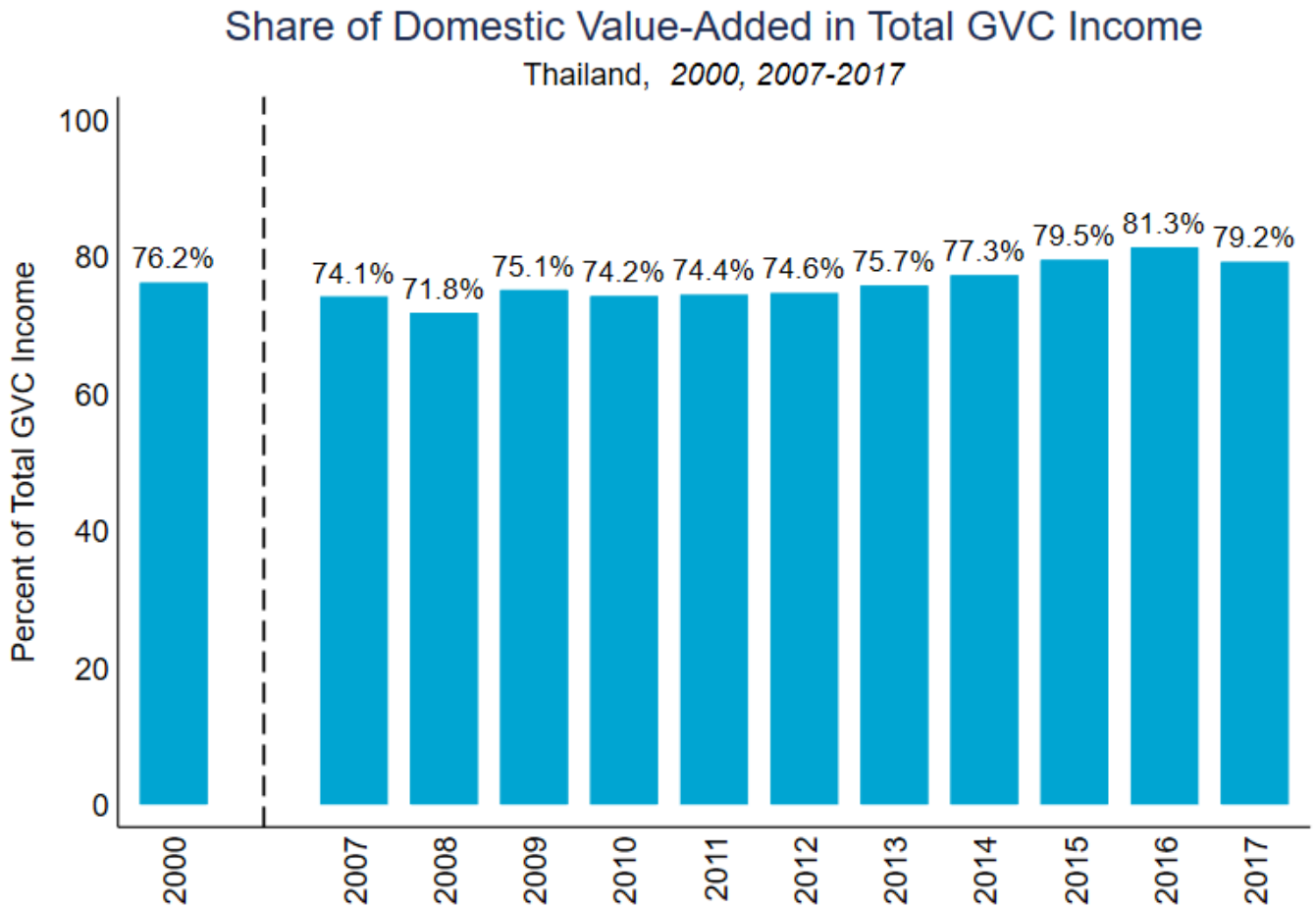
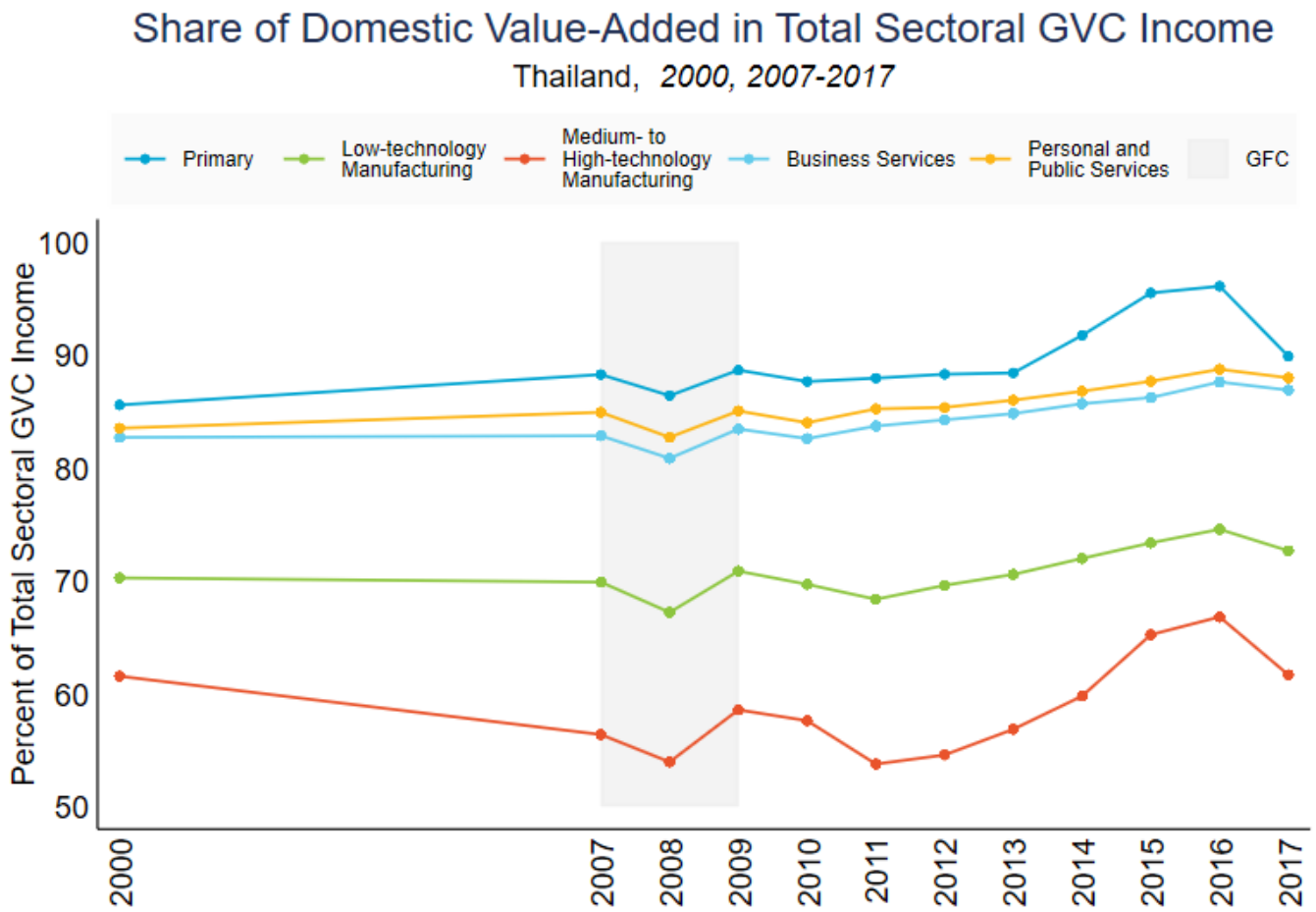
Share of domestically originating value added in total GVC income for Thailand is between 71% and 82%.



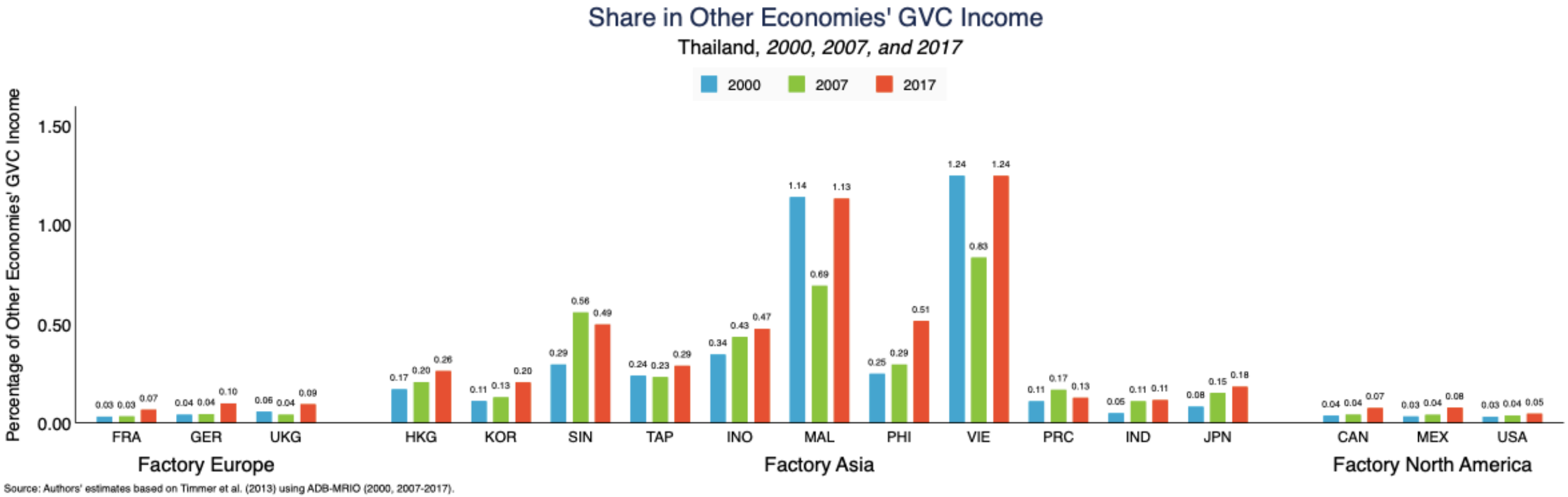
Source: Authors' estimates based on Timmer et al. (2013) using ADB-MRIO (2000, 2007-2017).

Domestic value added as a share of sectoral GVC income is highest for products that serve as inputs to production and for services.



Source: Authors' estimates based on Timmer et al. (2013) using ADB-MRIO (2000, 2007-2017).

Thailand's contribution to other economies' final products is highest in neighboring economies.



Source: Authors' estimates based on Timmer et al. (2013) using ADB-MRIO (2000, 2007-2017).

Decomposition of Gross Trade into Value-Added Terms

Koopman, R., Wang, Z., Wei, S.J. (2014). Tracing value-added and double counting in gross exports. *American Economic Review, 104*(2), 459–494. doi: 10.1257/aer.104.2.459

Wang, Z., Wei, S.J., & Zhu, K. (2018). *Quantifying international production sharing at the bilateral and sectoral levels* (NBER Working Paper 19677). Cambridge, MA: National Bureau of Economic Research. Retrieved from the National Bureau of Economic Research: https://www.nber.org/papers/w19677

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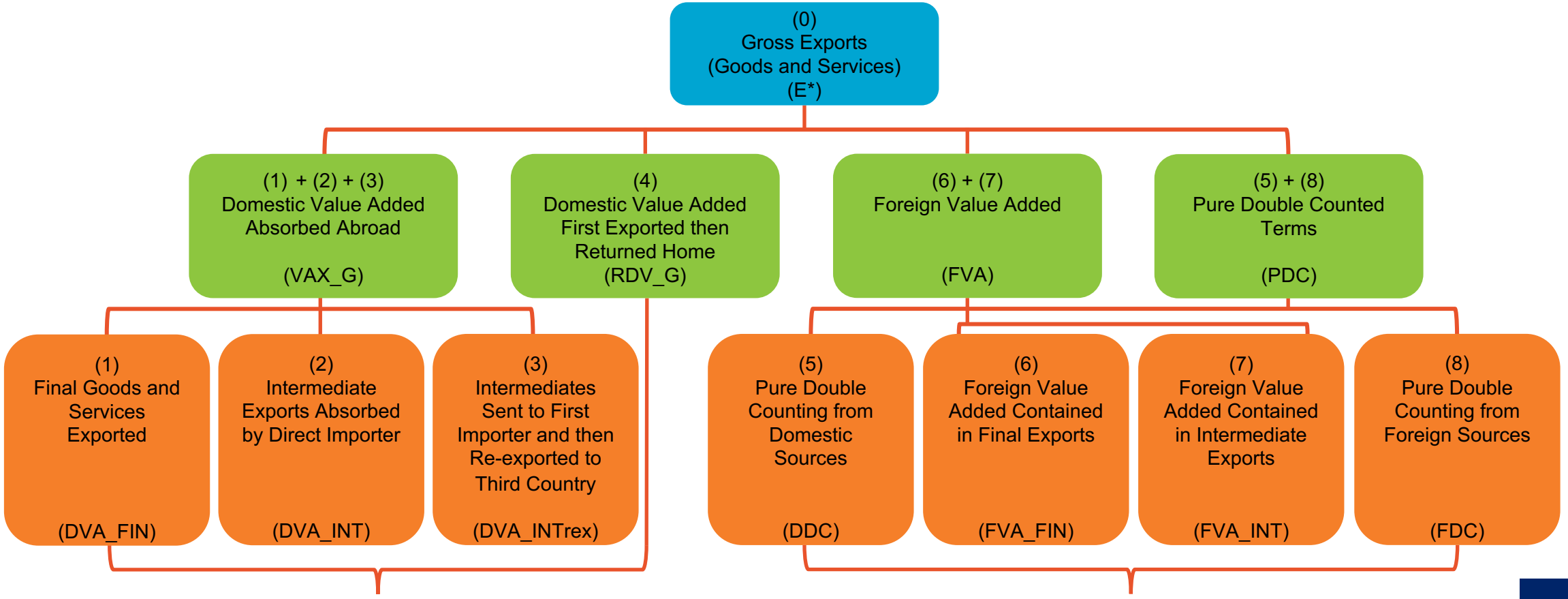
WWZ (2018)

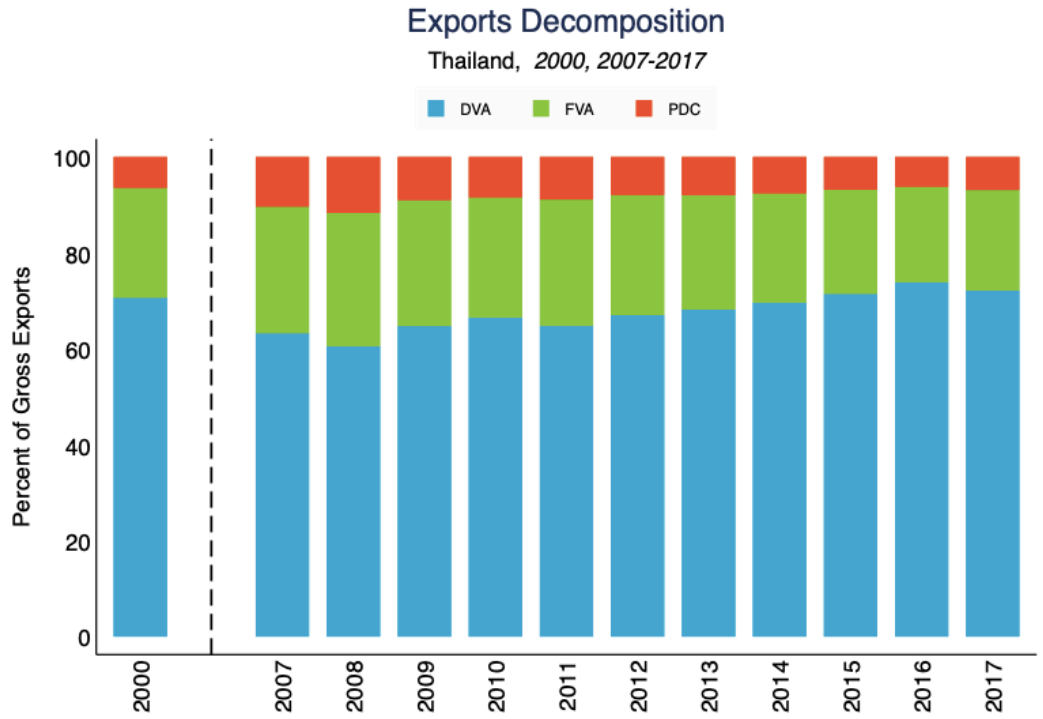
- Builds on Koopman, Wang, and Wei (2014)
  - Decomposes gross trade at the sector, bilateral, or bilateral-sector level.
  - Distinguishes between backward and forward linkages, allowing decomposition at a disaggregated level
  - Distinguishes two types of “trade in value-added” measures and two types of domestic value-added embedded in gross exports based on forward and backward industrial linkages at the country-sector or bilateral-sector levels

Gross Trade Accounting: Conceptual Framework

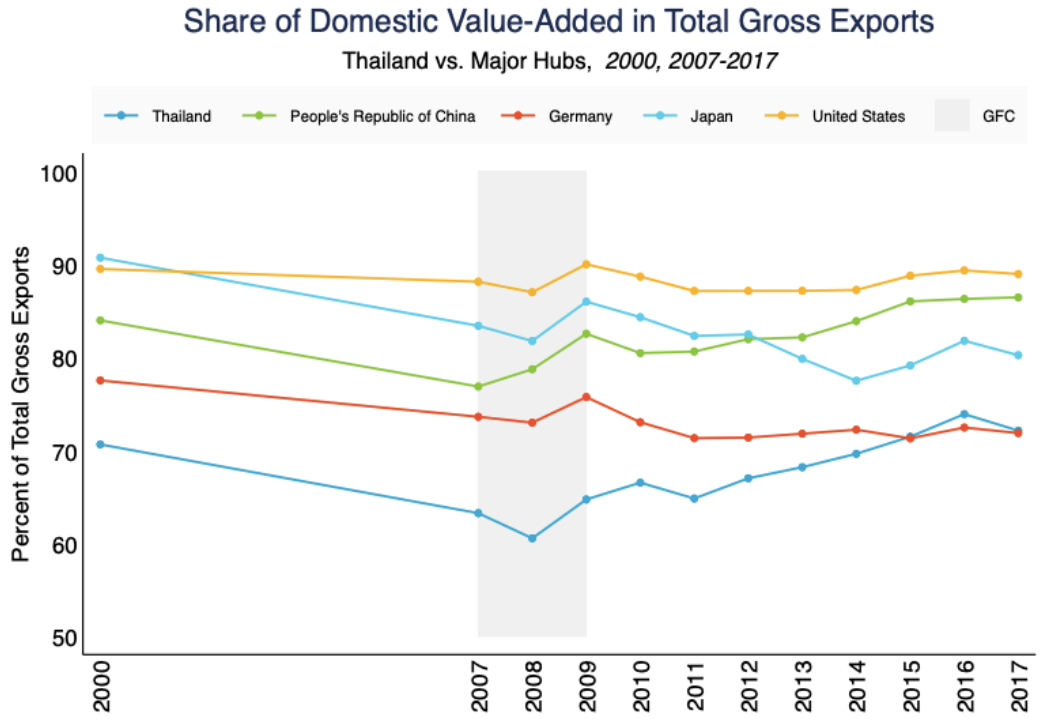


Note: E* can be at country-sector, country aggregate, bilateral-sector, or bilateral aggregate. Both VAX_G and RDV_G are based on backward industrial linkages.

Thailand's exports has a large share of domestic value added, but this is relatively lower compared to other GVC hubs.

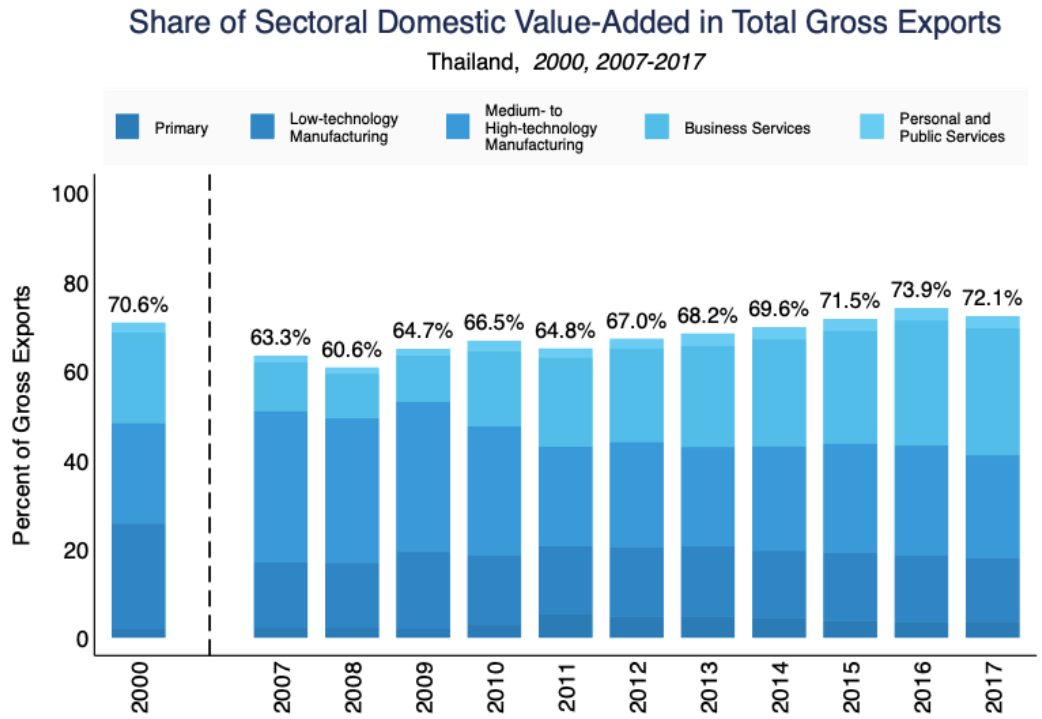


Source: Authors' estimates based on WWYZ (2018) using ADB-MRIO (2000, 2007-2017).

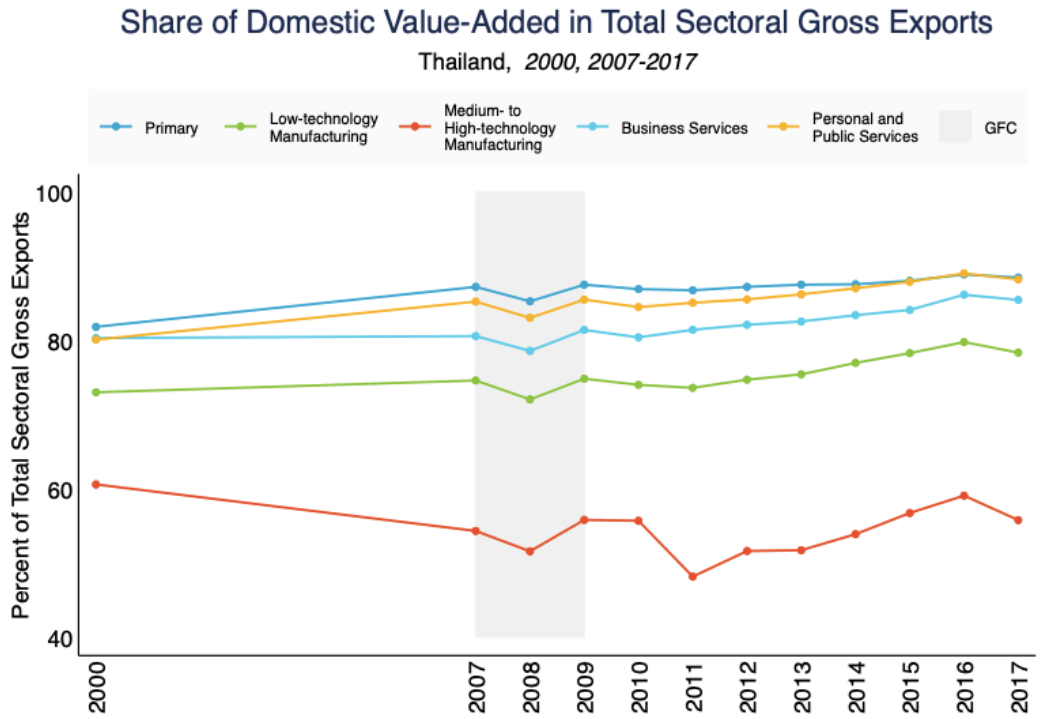


Source: Authors' estimates based on WWZ (2018) using ADB-MRIO (2000, 2007-2017).

Sector DVA contributions to Thailand's gross exports vary over time, but remain relatively flat when looking at sectoral gross exports.

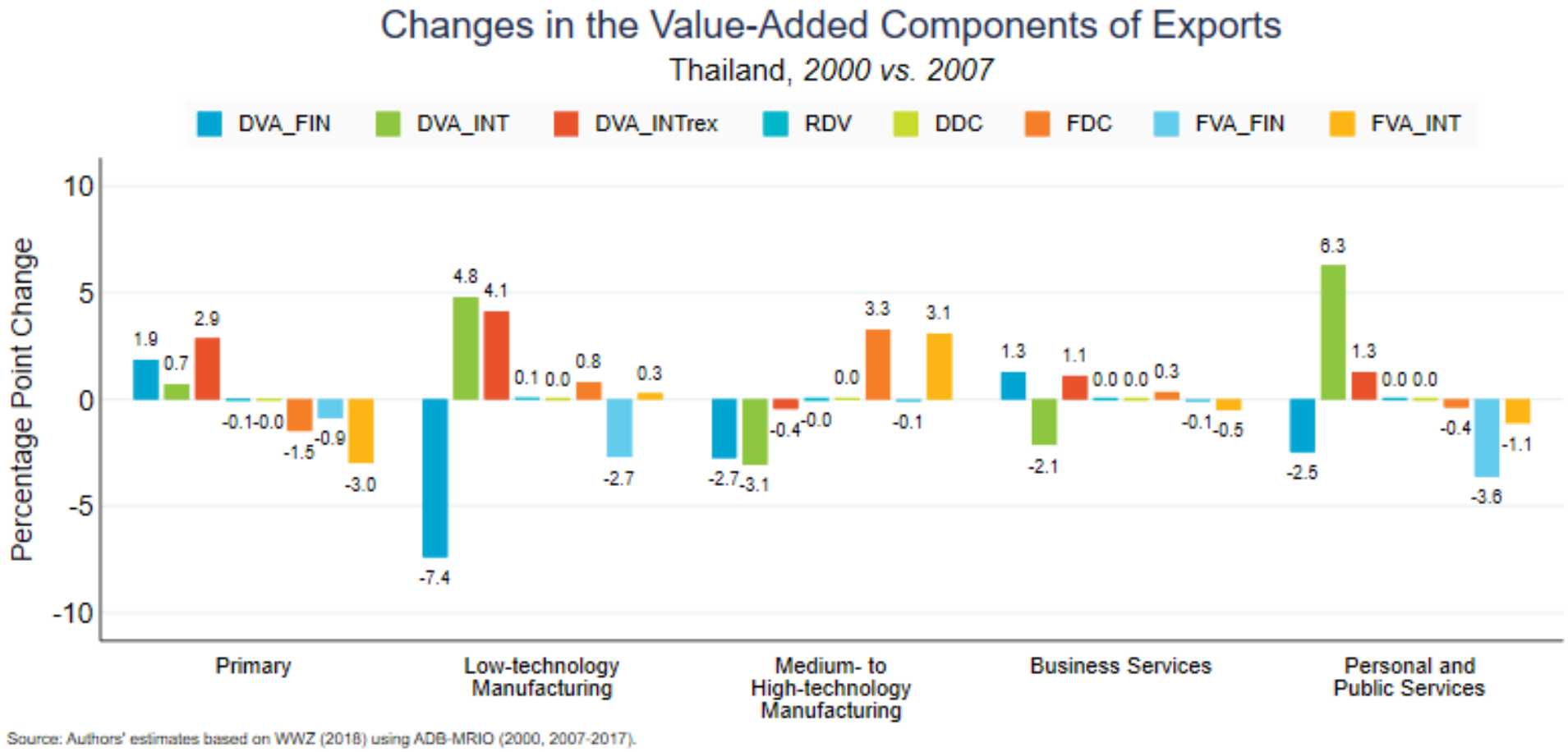


Source: Authors' estimates based on WWZ (2018) using ADB-MRIO (2000, 2007-2017).



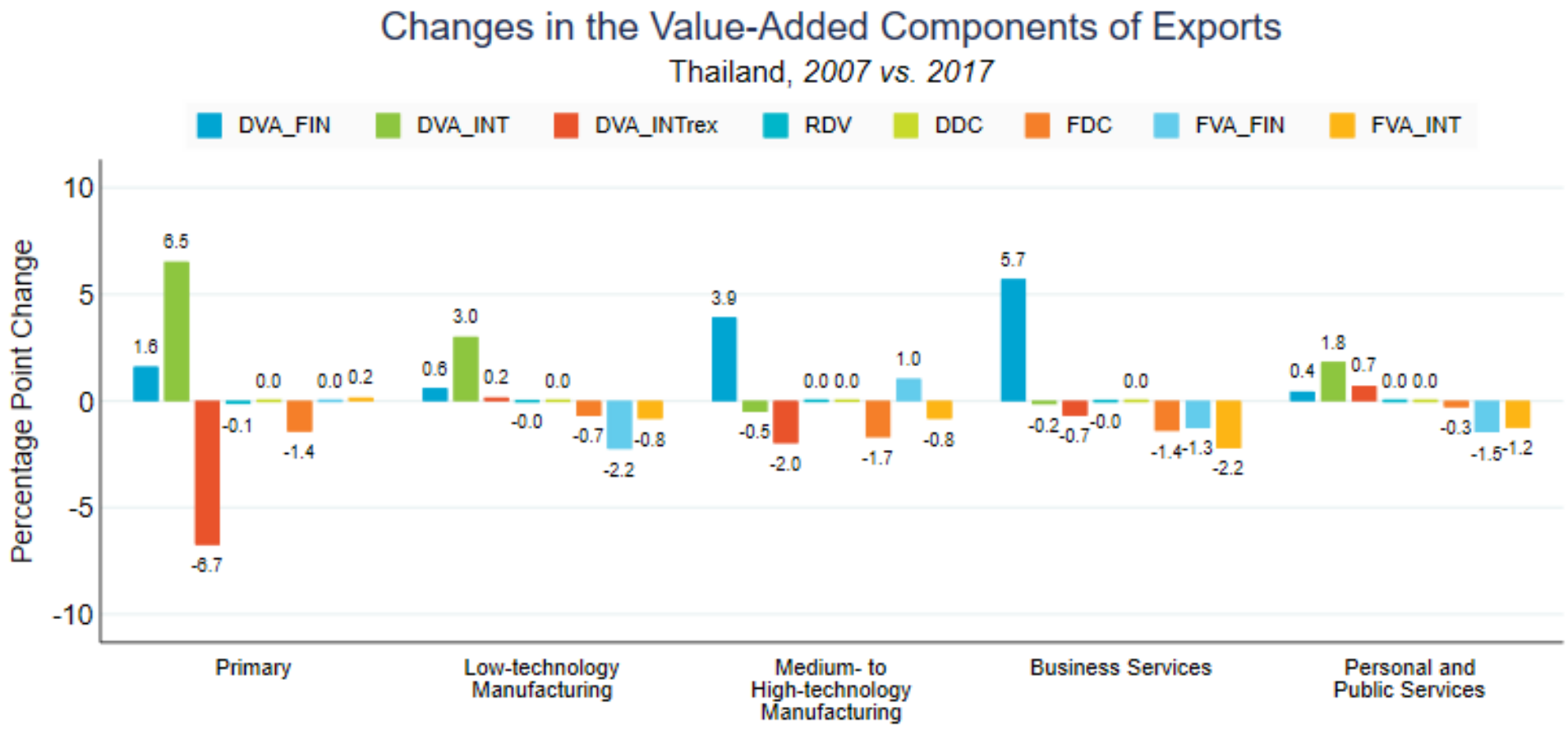
Source: Authors' estimates based on WWZ (2018) using ADB-MRIO (2000, 2007-2017).

Domestic value added embodied in exports for final consumption decreased for Thailand's manufacturing sectors as intermediate exports production increased.



Source: Authors' estimates based on WWZ (2018) using ADB-MRIO (2000, 2007-2017).

From 2007 to 2017, production has shifted more locally, with rising shares of domestic value-added components vs. foreign value-added components in total sectoral gross exports.



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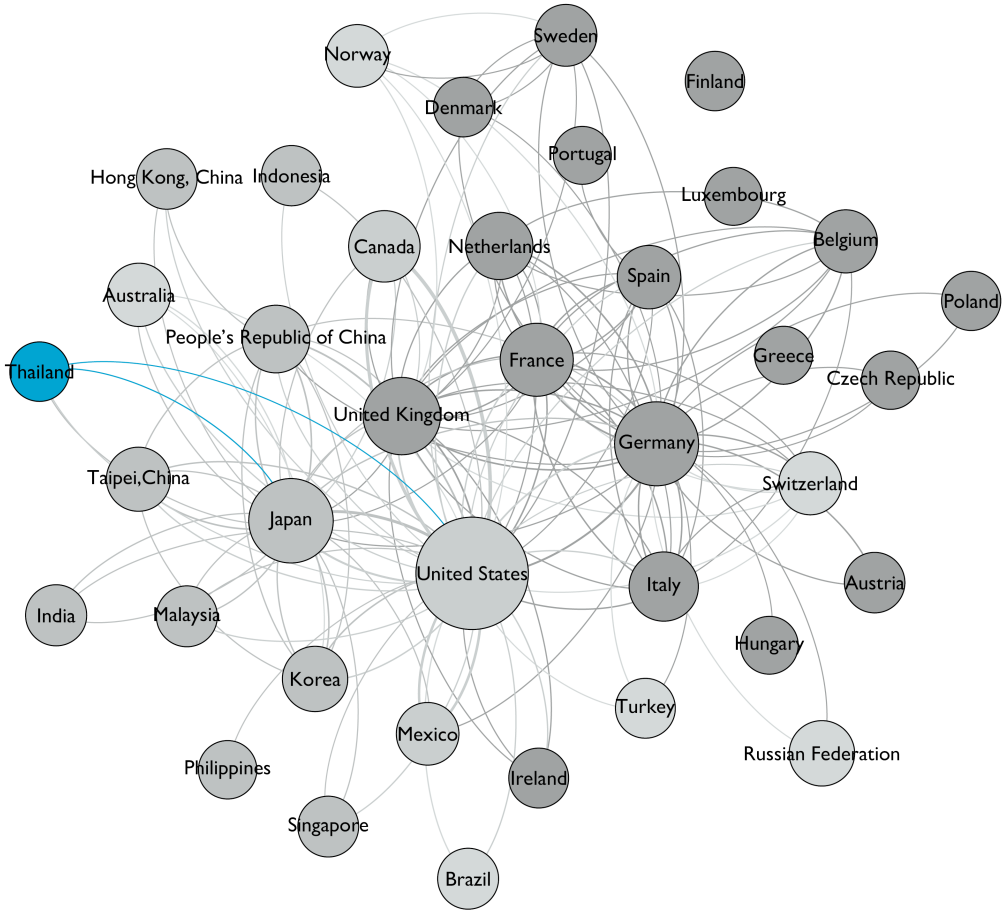
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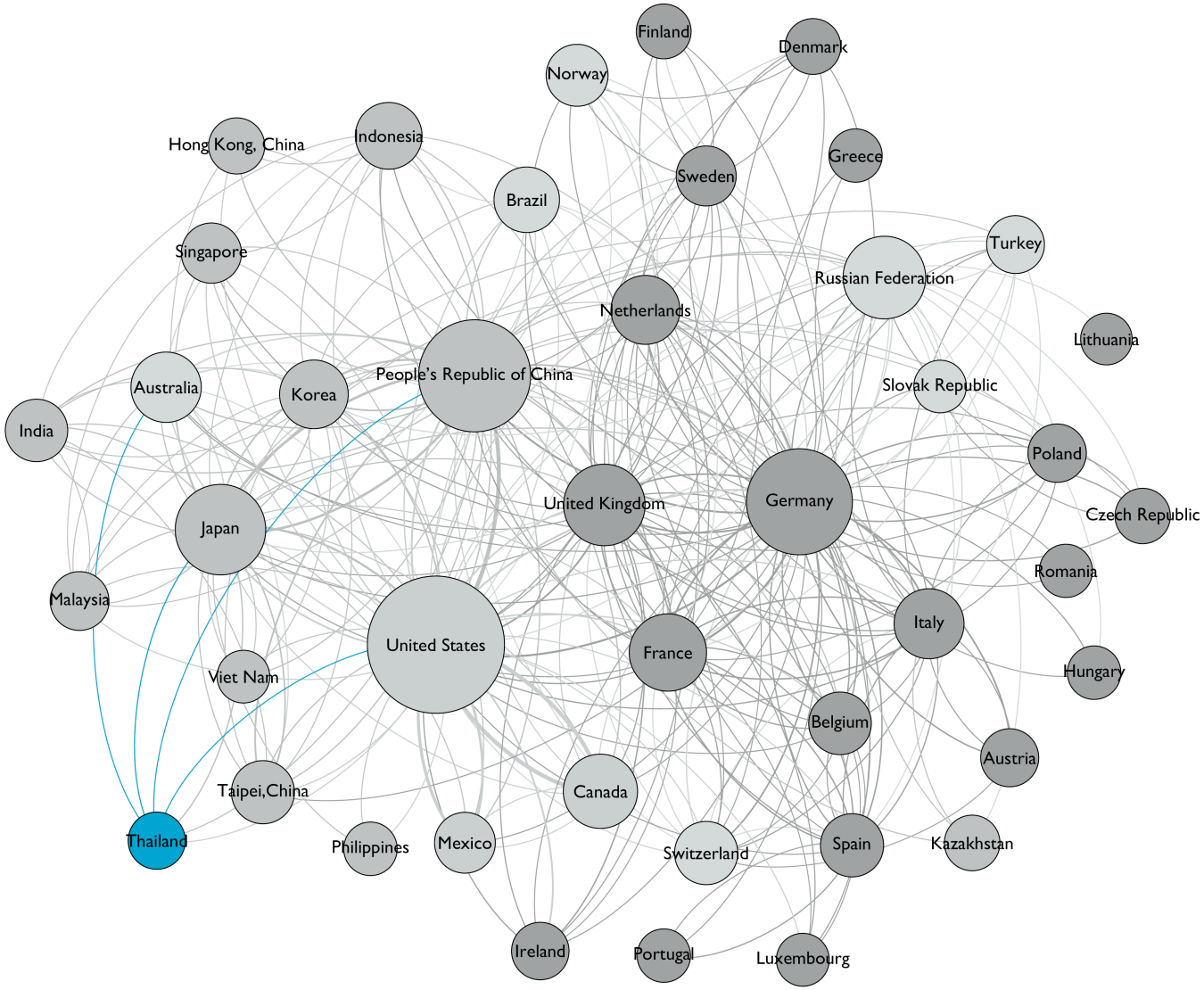
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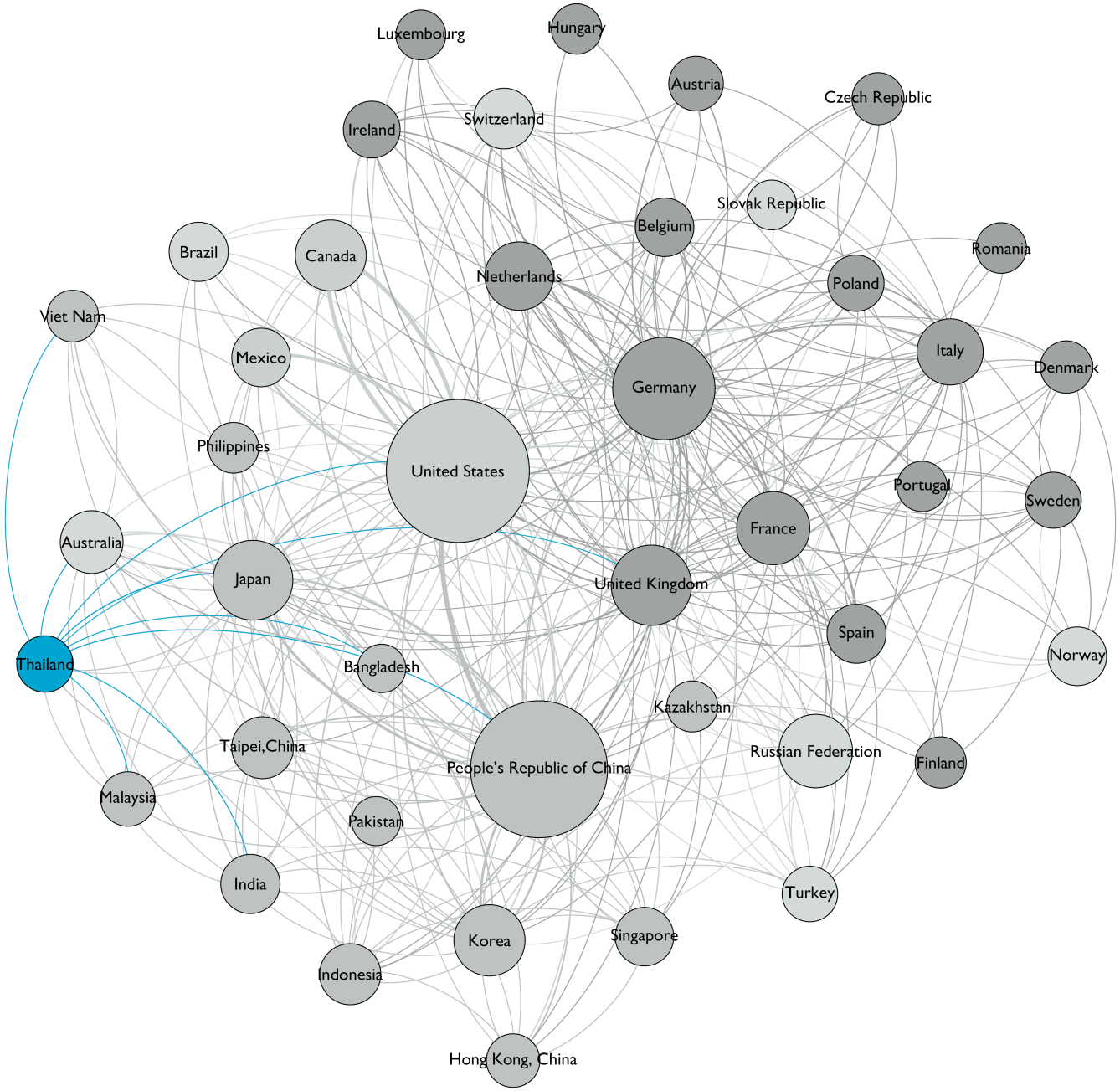
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Russian Federation
Finland
People's Republic of China
Malaysia
Pakistan
India
Turkey
Korea
Singapore
Indonesia
Hong Kong, China

2017

Revealed Comparative Advantage

DVA_F = VAX_F + RDV_F

- VAX_F measures the amount of domestic value added originating from a specific sector (e.g., from the US electronics sector), via all sectors' gross exports from the source country (i.e., including gross exports from the US automobile and machinery sectors in addition to the US electronics sector), and ultimately absorbed in a particular destination country.
- RDV_F is the amount of DVA from a specific sector embodied in the source country's intermediate gross exports to Country r, but eventually return to, via all possible routines through third countries and other sectoral linkages, and is absorbed in source Country s.
- DVA_F is the measure of DVA from a particular sector of the source Country s that is embodied in Country s's gross exports via forward industrial linkages.

Traditional Revealed Comparative Advantage

- **Traditional definition:** share of a country-sector's gross exports in the country's total gross exports divided by sector's gross exports from all countries as a share of world total gross exports.

$$TRCA_i^r = \left(\frac{e_i^{r*}}{\sum_{i=1}^{N} e_i^{r*}}\right) \Bigg/ \left(\frac{\sum_{k=1}^{G} e_i^{k*}}{\sum_{i}^{N}\sum_{k=1}^{G} e_i^{k*}}\right)$$

- If $TRCA_i^r$>1, then country r is said to have a revealed comparative advantage in sector i

Weaknesses of TRCA

- It ignores the fact that a country-sector's value added may be exported indirectly via the country's exports in other sectors
  - A conceptually correct measure should include indirect value added exports
- It ignores the fact that a country-sector's gross exports partly reflect foreign VA content
  - A conceptually correct measure should exclude foreign VA content

New Revealed Comparative Advantage

- **New definition:** share of a country-sector's forward-linkage based measure of DVA in exports in the country's total DVA in exports divided by that sector's total forward-linkage based DVA in exports as a share of global value added in exports

$$NRCA_i^r = \left(\frac{DVA\_F_i^{r*}}{\sum_{i=1}^{N} DVA\_F_i^{r*}}\right) \Bigg/ \left(\frac{\sum_{k=1}^{G} DVA\_F_i^{k*}}{\sum_{i}^{N}\sum_{k=1}^{G} DVA\_F_i^{k*}}\right)$$

- If $NRCA_i^r$>1, then country r is said to have a revealed comparative advantage in industry i

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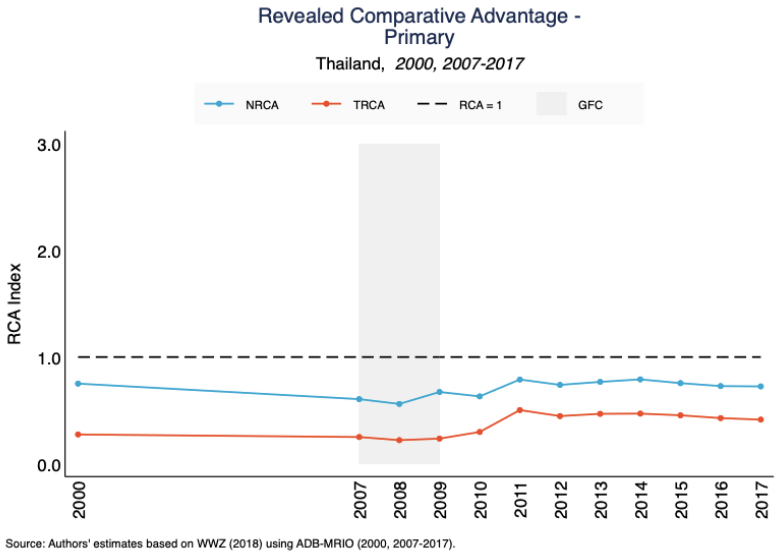
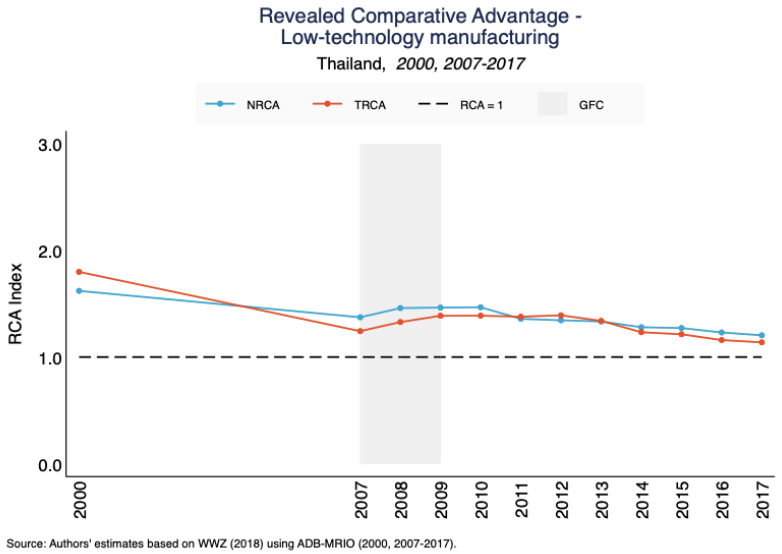
NRCA and TRCA because of the inclusion of indirect value added exports and the exclusion of foreign value added components.



Source: Authors' estimates based on WWZ (2018) using ADB-MRIO (2000, 2007-2017).



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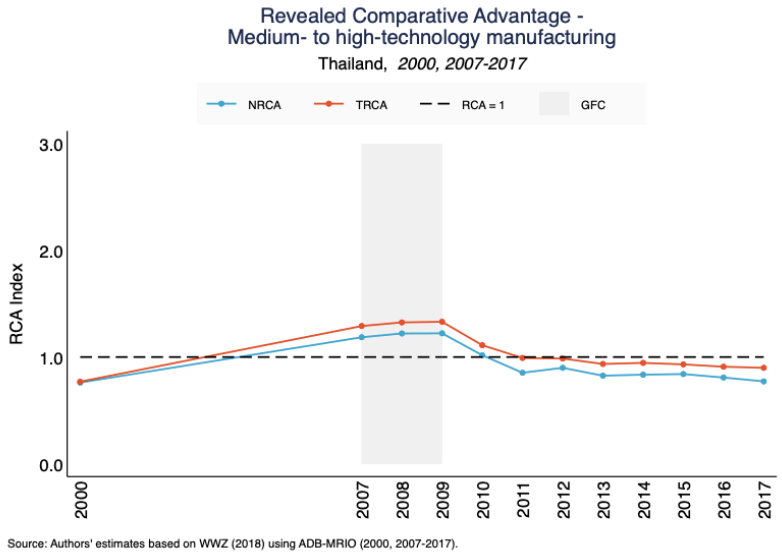
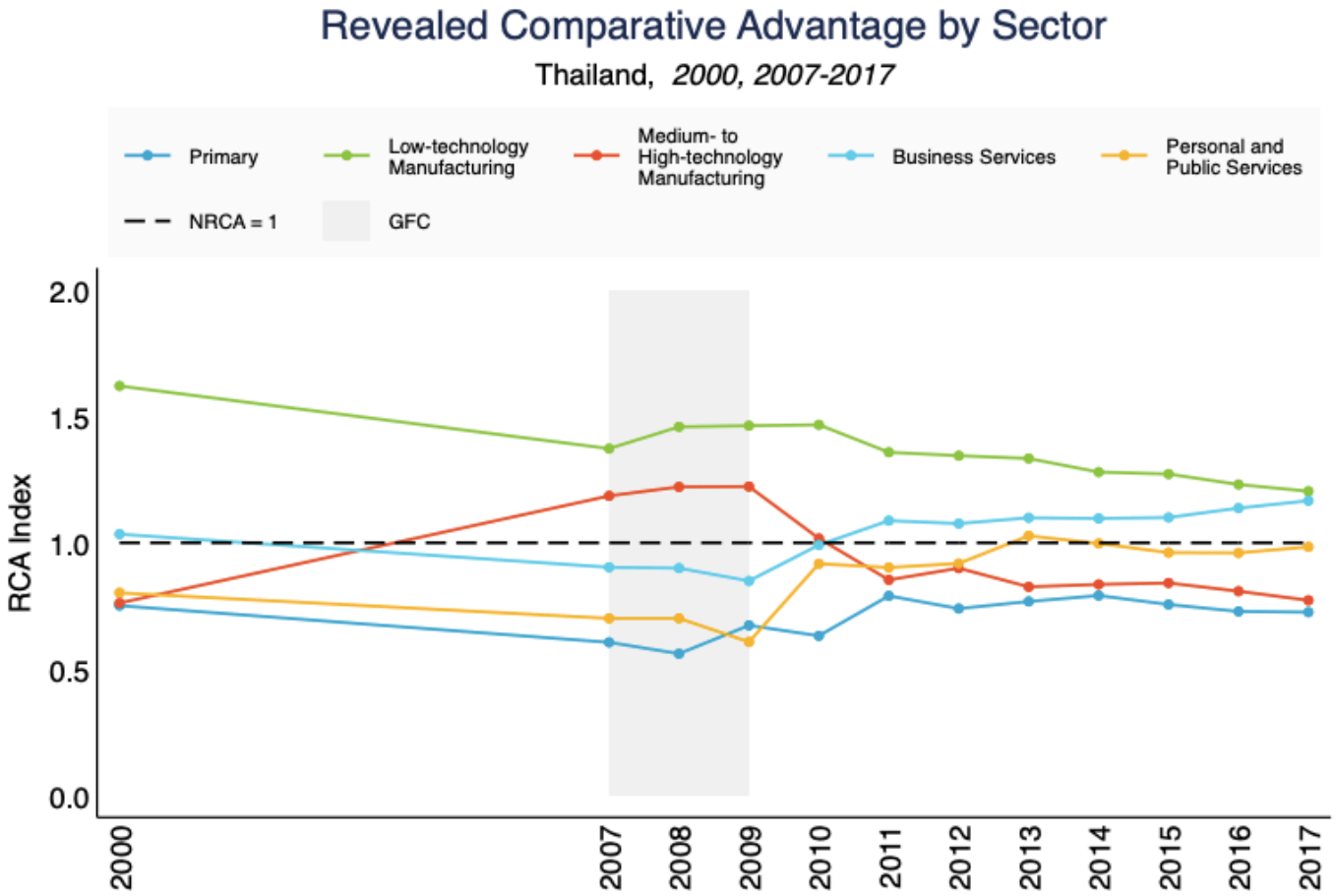


Source: Authors' estimates based on WWZ (2018) using ADB-MRIO (2000, 2007-2017).

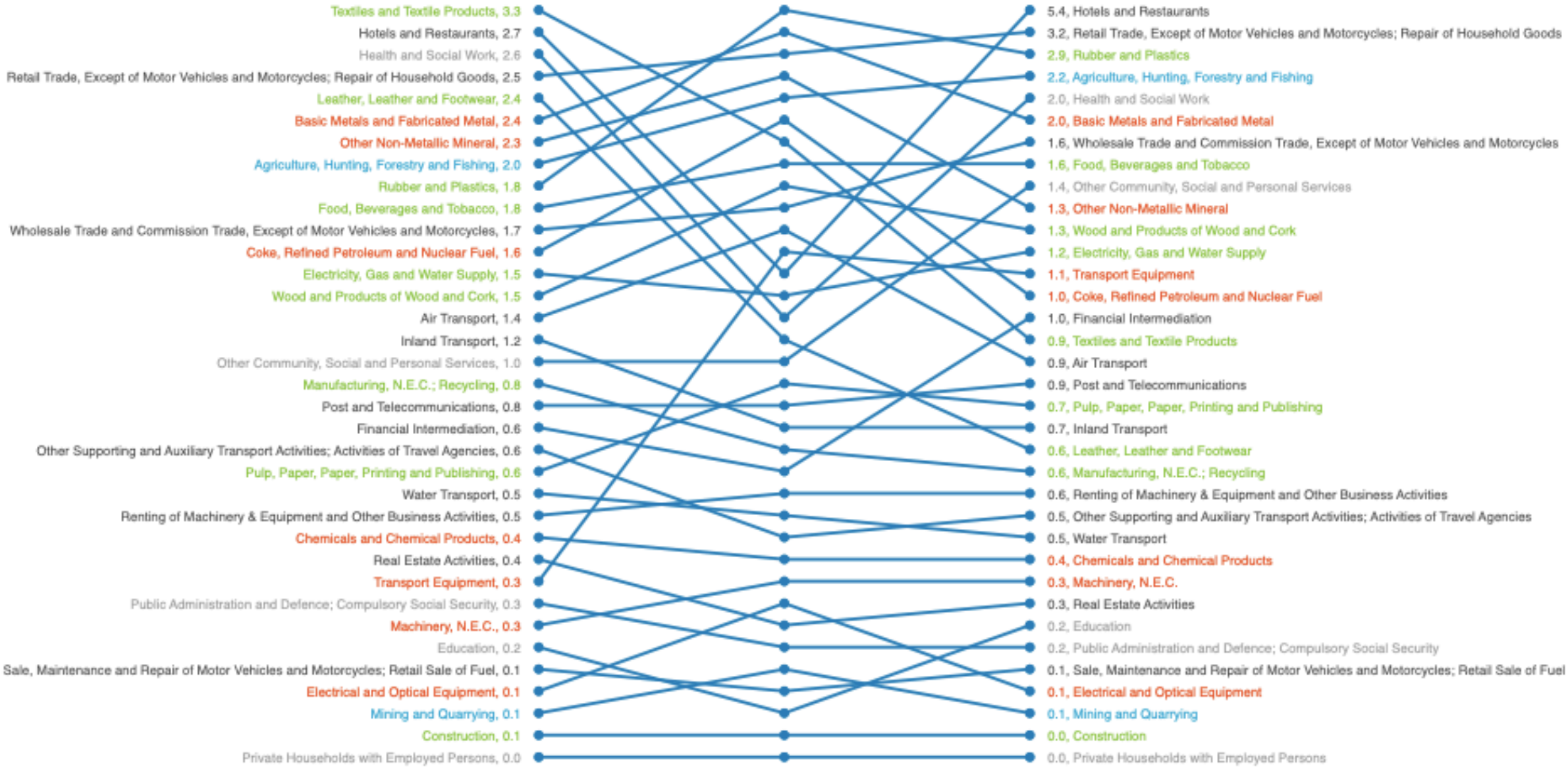
Thailand has a comparative advantage in low-technology manufacturing and business services.



Source: Authors' estimates based on WWZ (2018) using ADB-MRIO (2000, 2007-2017).

Sectoral NRCA Ranking

Thailand, *2000, 2007, and 2017*



Participation in Global Value Chains

Wang, Z. Wei, S.J., Yu, X. Zhu, K. (2017). *Measures in participation in global value chains and global business cycles (*NBER Working Paper 23222). Cambridge, MA: National Bureau of Economic Research. Retrieved from the National Bureau of Economic Research: https://www.nber.org/ papers/w23222

Decomposition of Value Added and Final Goods Production

$$Va' = \hat{V}BY = \underbrace{\hat{V}LY^D}_{(1)V\_D} + \underbrace{\hat{V}LY^F}_{(2)V\_RT} + \underbrace{\hat{V}LA^FLY^D}_{(3a)V\_GVC\_S} + \underbrace{\hat{V}LA^F(BY - LY^D)}_{(3b)V\_GVC\_C}$$

$$Y' = VB\hat{Y} = \underbrace{VL\hat{Y}^D}_{(1)Y\_D} + \underbrace{VL\hat{Y}^F}_{(2)Y\_RT} + \underbrace{VLA^FL\hat{Y}^D}_{(3a)Y\_GVC\_S} + \underbrace{VLA^F(B\hat{Y} - L\hat{Y}^D)}_{(3b)Y\_GVC\_C}$$

Decomposition of GDP by Country-Sector

(Which types of production and trade are GVC activities?)

A country-sector's total value added (V)
GDP by industry

- **0** In production of final products to domestic market directly (V_D)
- **1** In production of final exports directly (V_RT)
- In production of intermediate exports (V_GVC)
  - **1** Absorbed by direct importer Simple GVCs (V_GVC_S)
  - **≥2** Re-export/re-import Complex GVCs (V_GVC_S)

Decomposition of Final Goods Production by Country-Sector

(Which part of final goods production and trade belong to GVCs?)

Production of final goods and services by country-sector (Y)
Output by industry

- **0** Domestic VA in domestically used final products (Y_D)
- **1** Domestic VA in final exports (Y_RT)
- Domestic and foreign VA in intermediate imports (Y_GVC)
  - **1** Partner VA in production of domestic used products (V_GVC_S)
  - **≥2** In production of exported products (Y_GVC_S)

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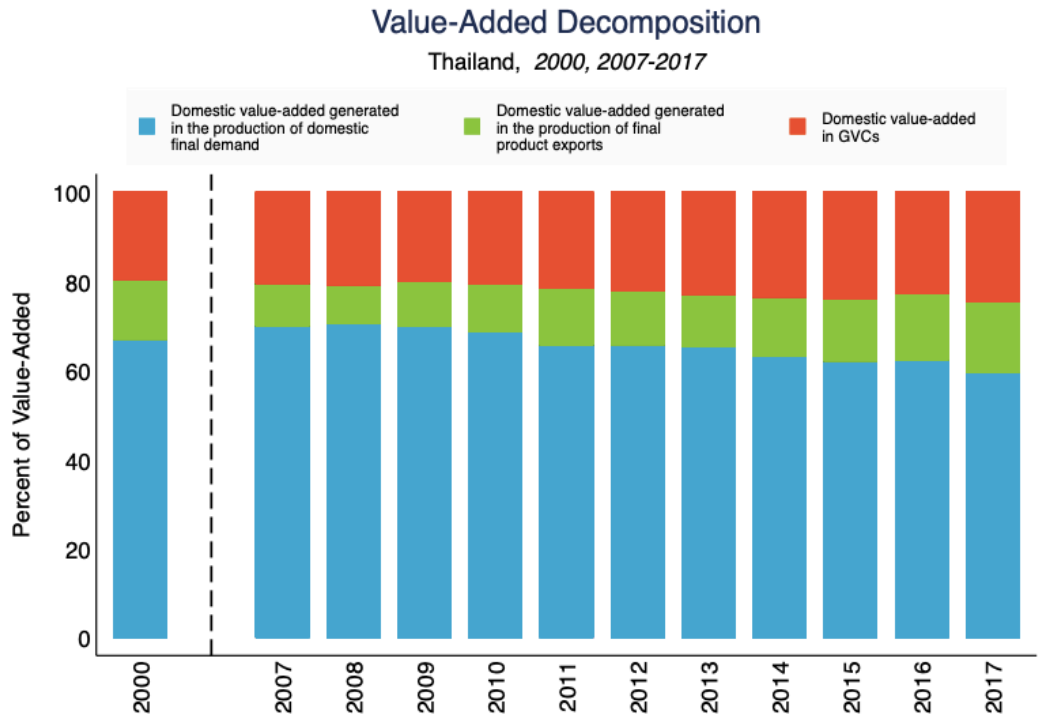
An increasing portion of value added is going to GVC-related activities, while a lower share of value added in final goods production comes from GVC-related activities.



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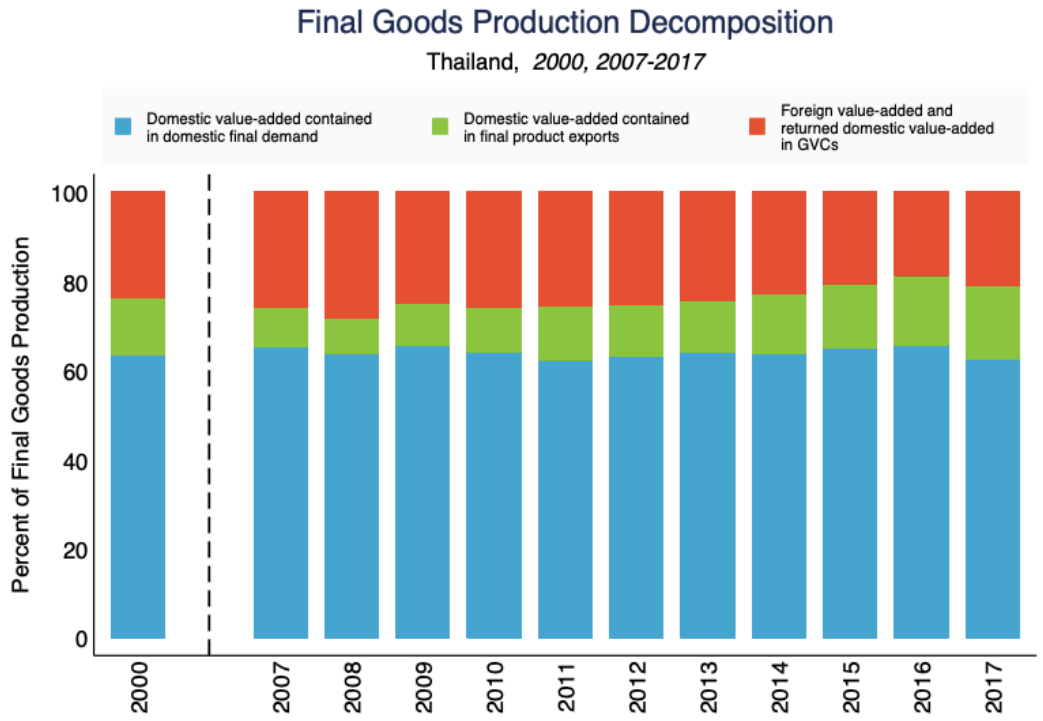


Source: Authors' estimates based on WWYZ (2018) using ADB-MRIO (2000, 2007-2017).

Global Value Chain Participation Index

Forward Participation

$$GVCPt\_f = \frac{V\_GVC}{Va'} = \frac{V\_GVC\_S}{Va'} + \frac{V\_GVC\_C}{Va'}$$

- What percentage of production factors employed in a country-sector has been involved in cross country production sharing activity?
- Denominator is the total value added generated in production from the country-sector; numerator is the total domestic value added of that country-sector that is embodied in its intermediate exports to the world.
- Domestic value added generated from GVCs production and trade activities a share of total sector value added (GDP)

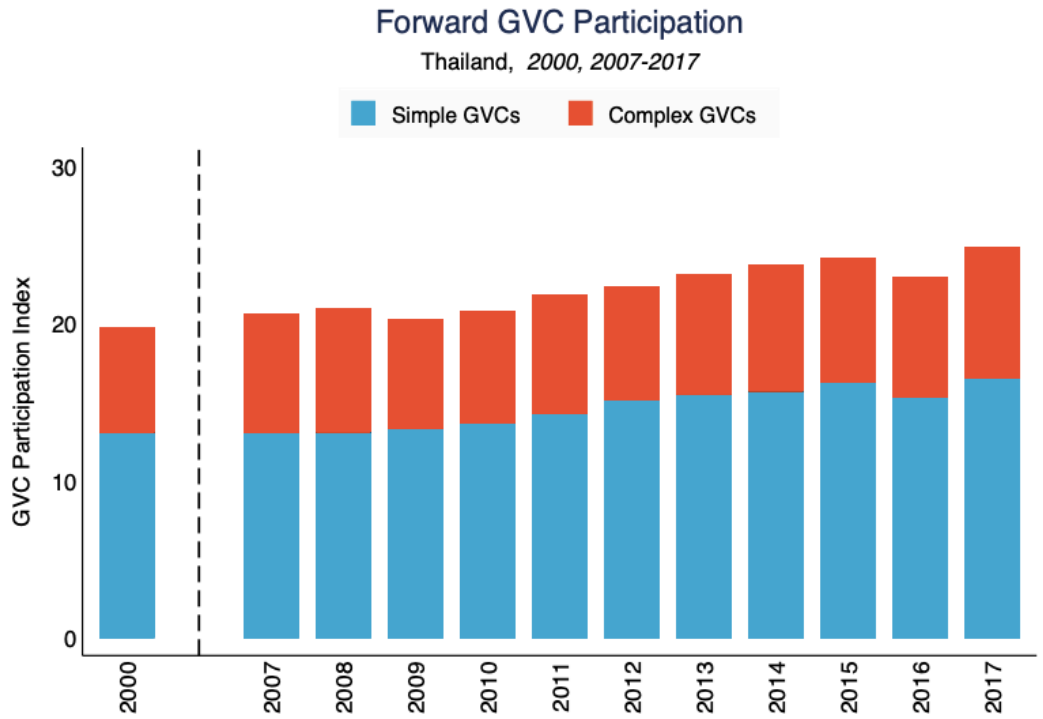
Backward Participation

$$GVCPt\_b = \frac{Y\_GVC}{Y} = \frac{Y\_GVC\_S}{Y} + \frac{Y\_GVC\_C}{Y}$$

- What percentage of final products produced by a country-sector comes from GVC activities?
- It includes not only foreign value-added embodied in intermediate exports, but also domestic factor content that has returned home through international trade to satisfy domestic final demand.
- The percentage of a country's final goods production contributed by both domestic and foreign factors that involve cross country production sharing activities

Note: $Va' = Y$ in the world aggregate.

Thailand is becoming more actively engaged in upstream production activities in GVC when considering the whole economy.



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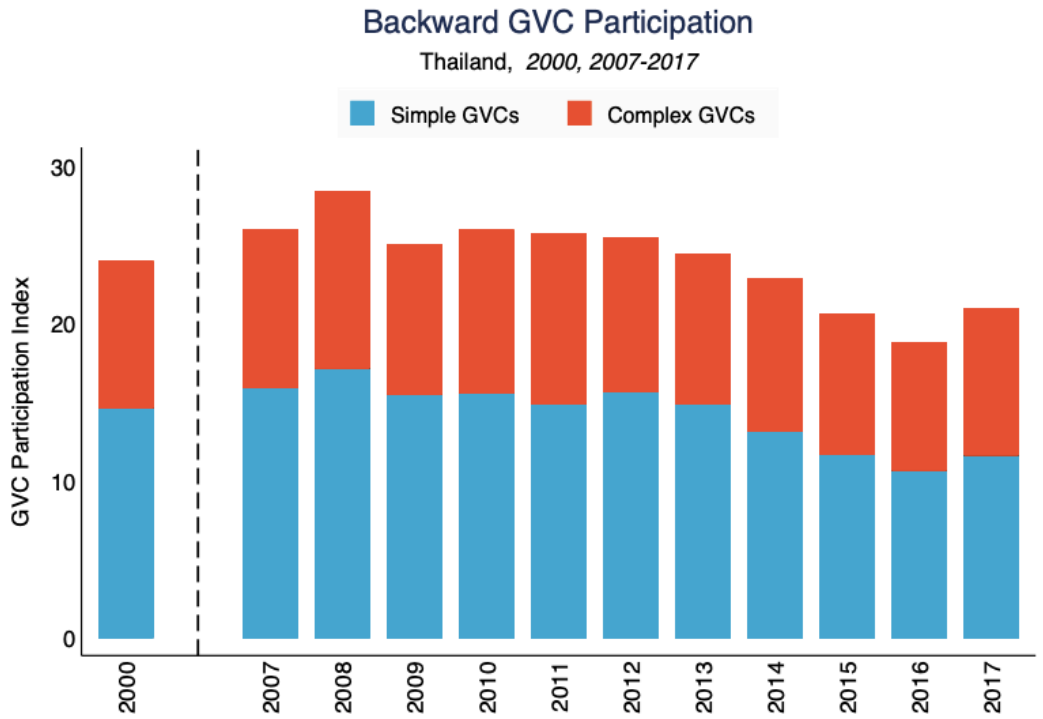
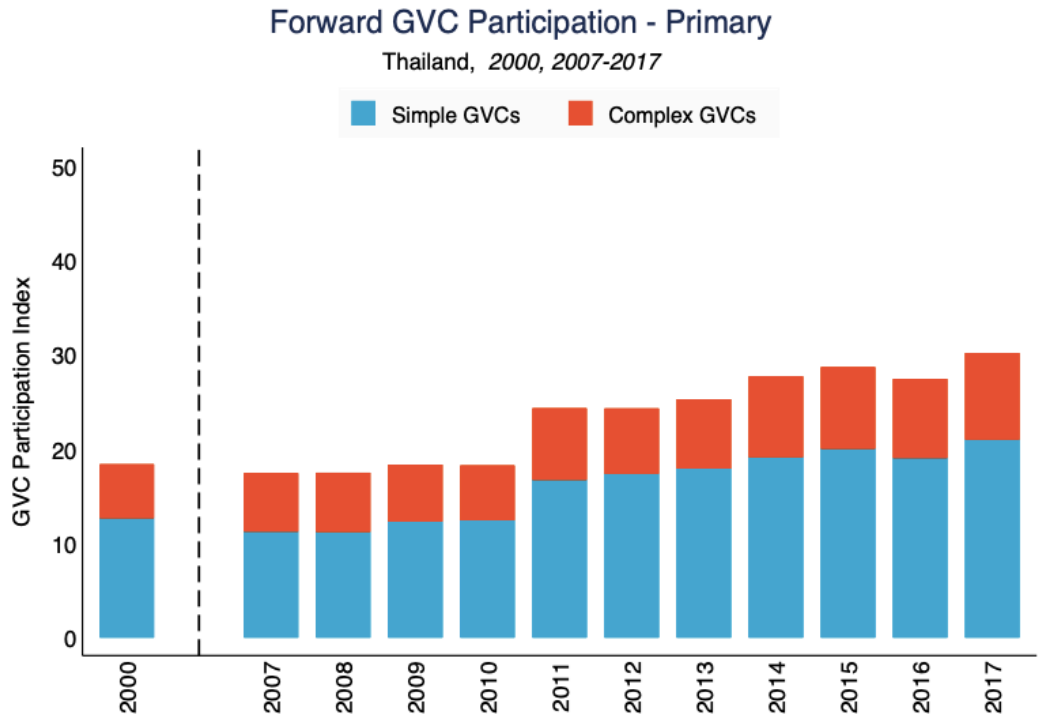
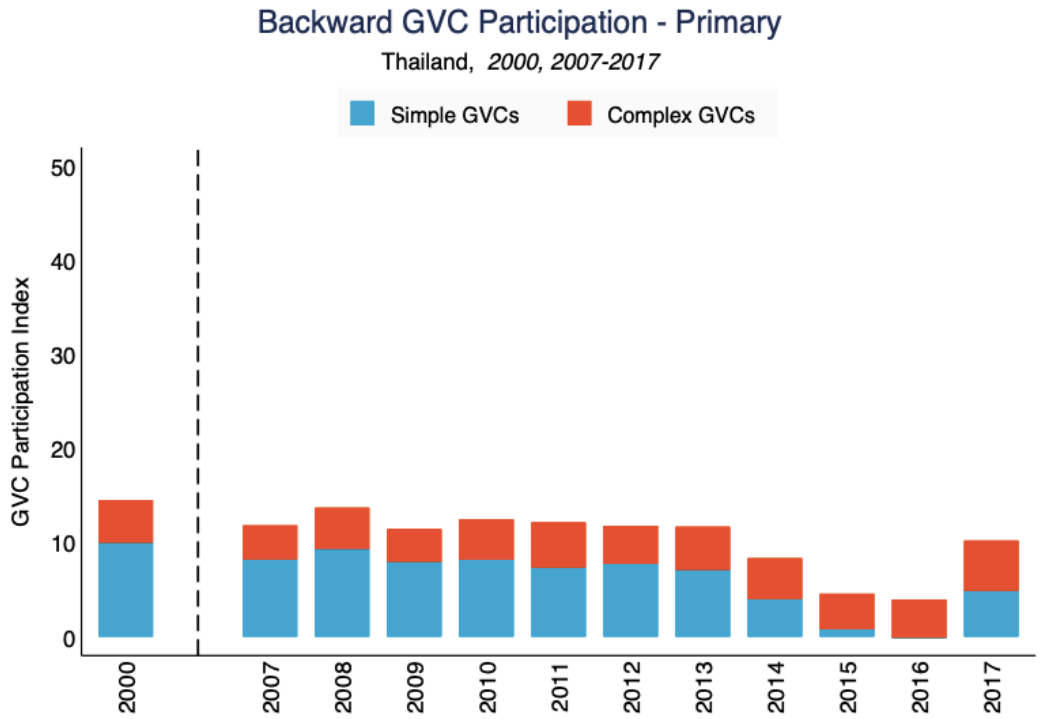


Source: Authors' estimates based on WWYZ (2018) using ADB-MRIO (2000, 2007-2017).

Thailand's primary sector is increasingly more engaged in upstream production activities in GVC.

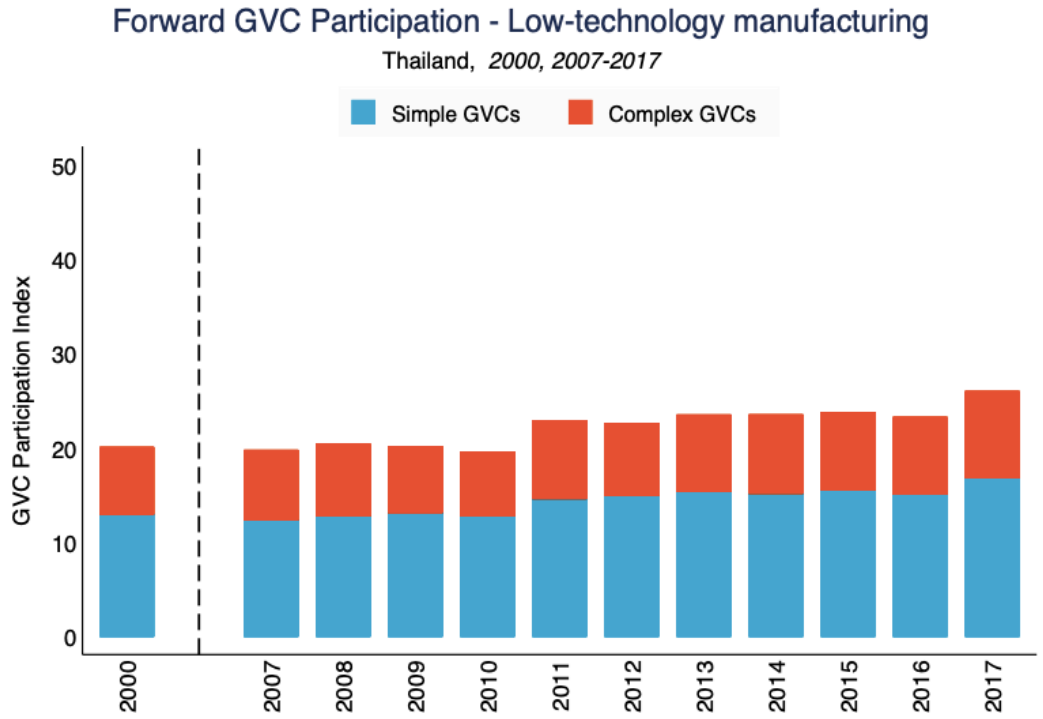


Source: Authors' estimates based on WWYZ (2018) using ADB-MRIO (2000, 2007-2017).

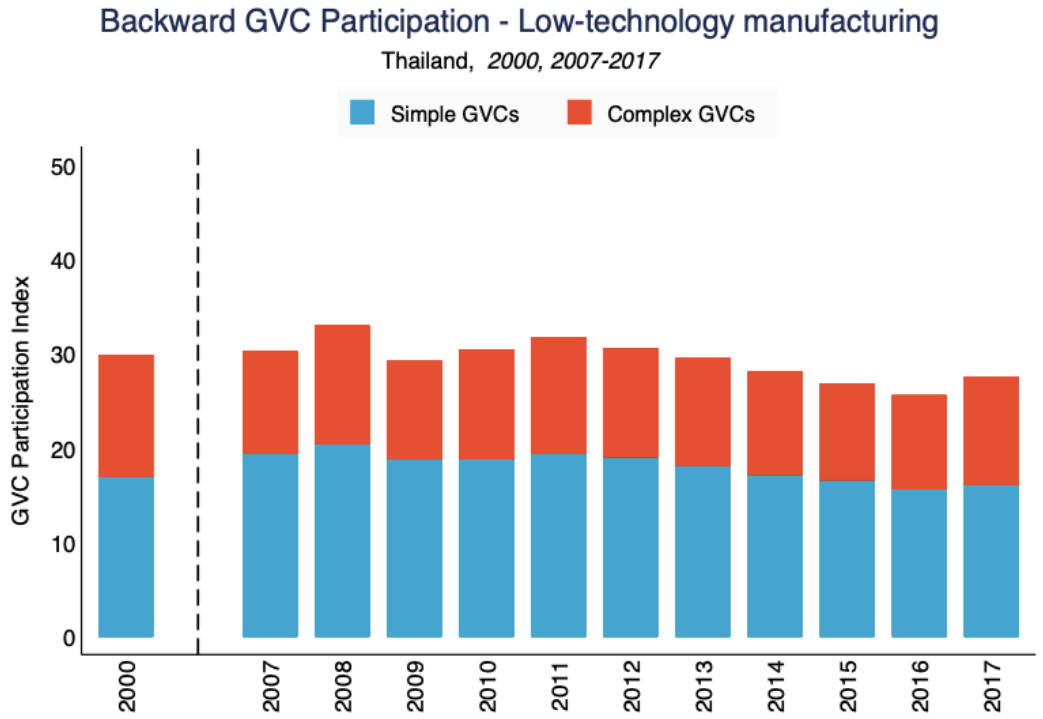


Source: Authors' estimates based on WWYZ (2018) using ADB-MRIO (2000, 2007-2017).

Higher backward participation in low-technology manufacturing suggests more engagement in downstream GVC activities.

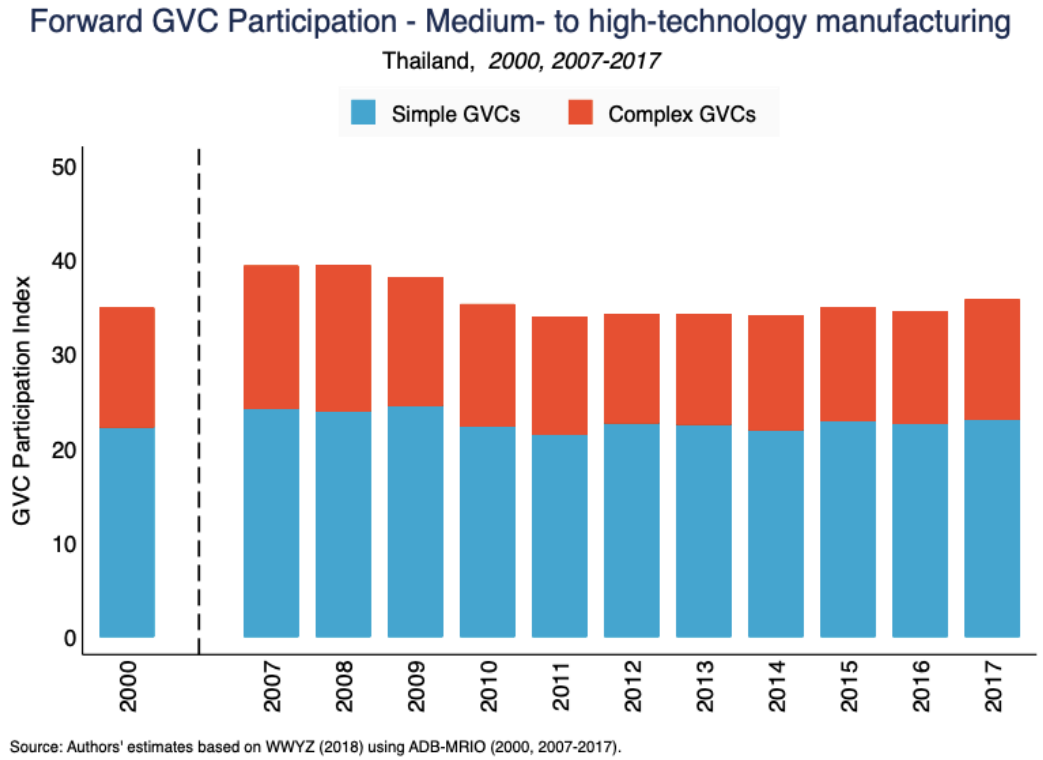


Source: Authors' estimates based on WWYZ (2018) using ADB-MRIO (2000, 2007-2017).

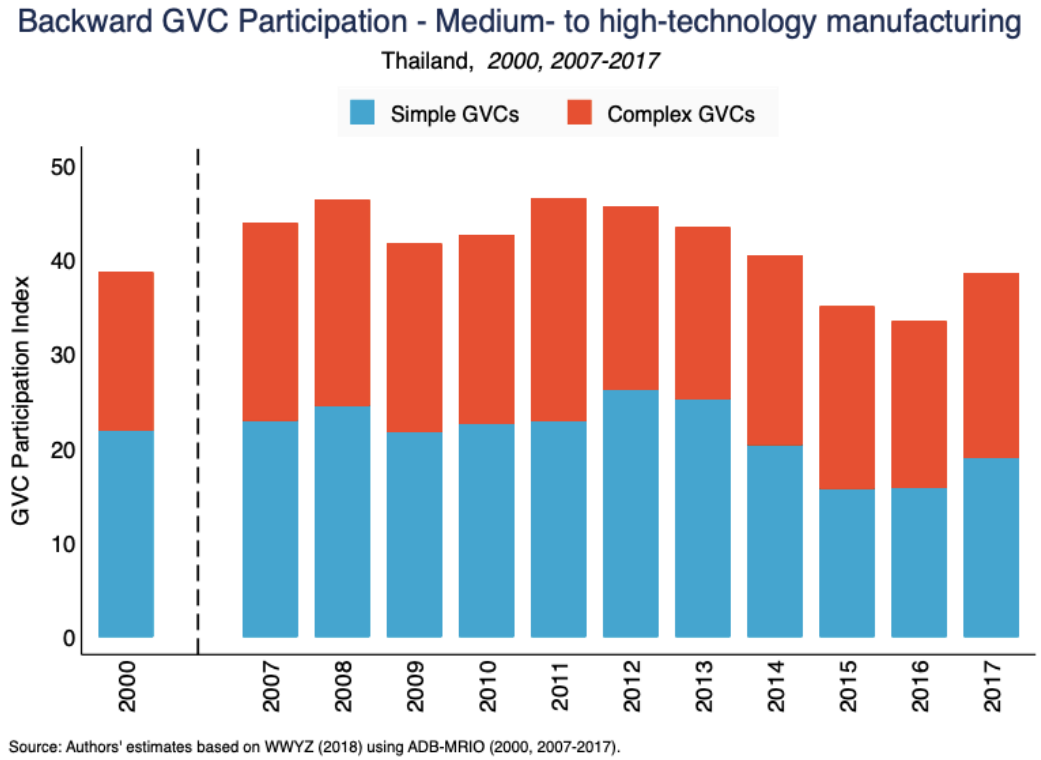


Source: Authors' estimates based on WWYZ (2018) using ADB-MRIO (2000, 2007-2017).

For the medium- to high-technology manufacturing sector, Thailand seems to be more engaged in downstream activities and engagement in simple and complex GVC activities is more equal in the backward linkage.

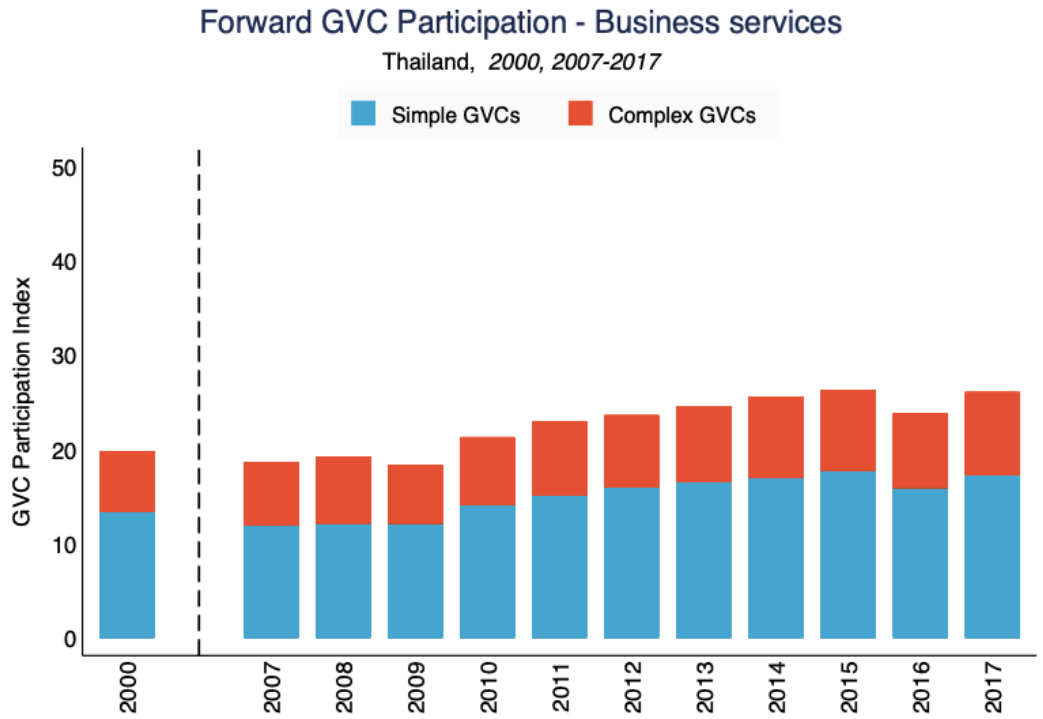


Source: Authors' estimates based on WWYZ (2018) using ADB-MRIO (2000, 2007-2017).

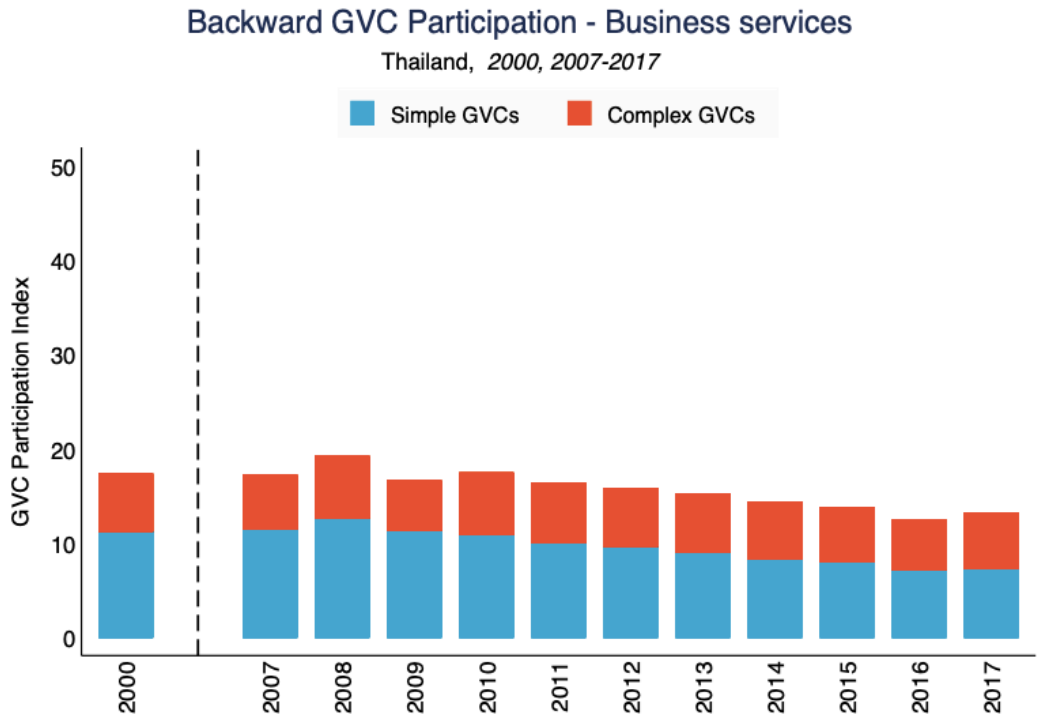


Source: Authors' estimates based on WWYZ (2018) using ADB-MRIO (2000, 2007-2017).

For the business services sector, the pattern is more or less the same as the aggregate economy.

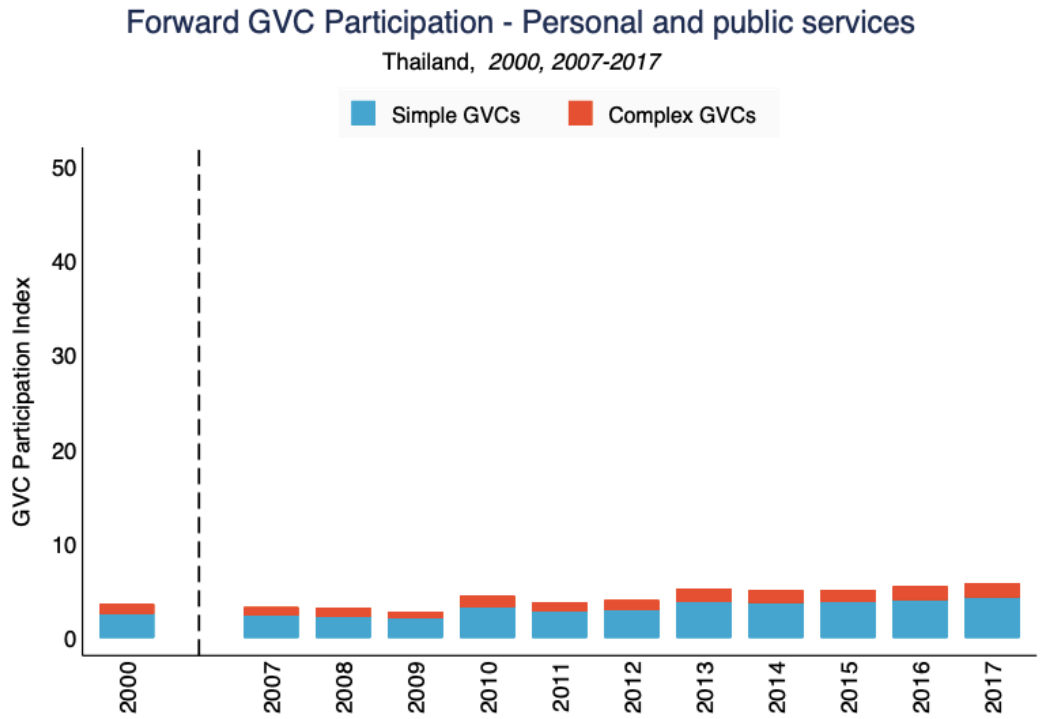


Source: Authors' estimates based on WWYZ (2018) using ADB-MRIO (2000, 2007-2017).

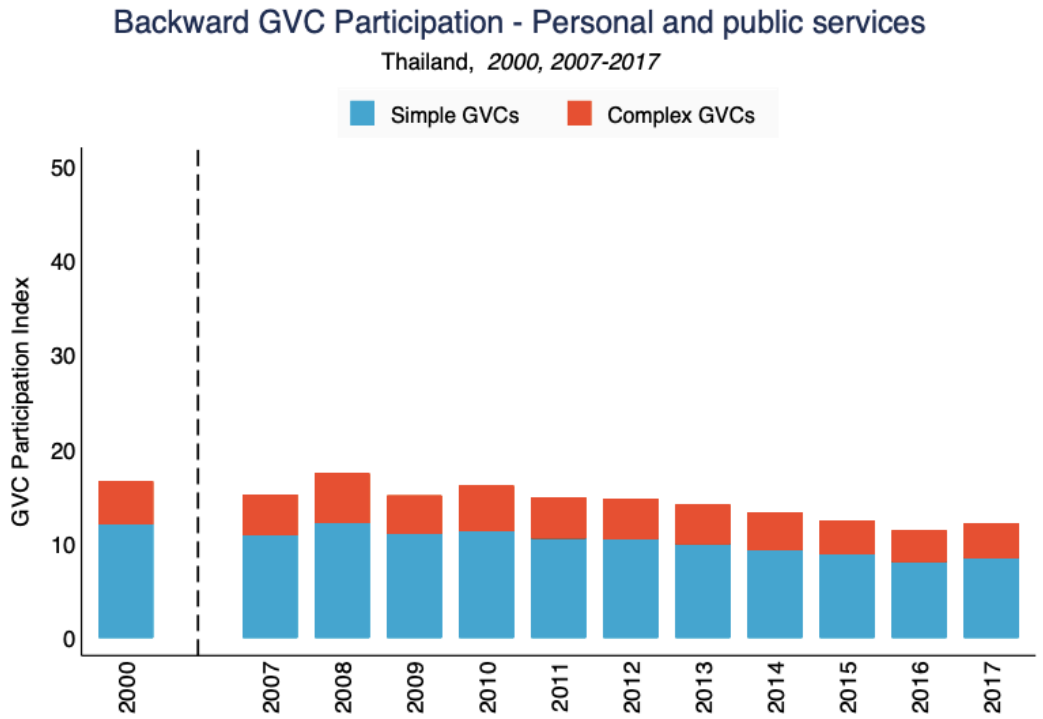


Source: Authors' estimates based on WWYZ (2018) using ADB-MRIO (2000, 2007-2017).

For the personal and public services sector: GVCPt_F < GVCPt_B



Source: Authors' estimates based on WWYZ (2018) using ADB-MRIO (2000, 2007-2017).



Source: Authors' estimates based on WWYZ (2018) using ADB-MRIO (2000, 2007-2017).

Upstreamness

Antràs, P., Chor, D., Fally, T., and Hillberry, R. (2012). Measuring the upstreamness of production and trade flows. American Economic Review: Papers & Proceedings, 102(3), 412-416.

Antràs, P. and Chor, D. (2017).

Upstreamness

$$U_i^r = 1 \cdot \frac{Y_i^r}{X_i^r} + 2 \cdot \frac{\sum_{s=1}^{S}\sum_{j=1}^{J} a_{ij}^{rs} Y_j^s}{X_i^r} + 3 \cdot \frac{\sum_{s=1}^{S}\sum_{j=1}^{J}\sum_{t=1}^{S}\sum_{k=1}^{J} a_{ij}^{rs} a_{jk}^{st} Y_k^t}{X_i^r} + \cdots$$

- It is clear that $U_i^r \geq 1$ and that larger values are associated with relatively higher levels of upstreamness of the output originating from sector r in country i.
- The upstreamness of each sector to the final market is:

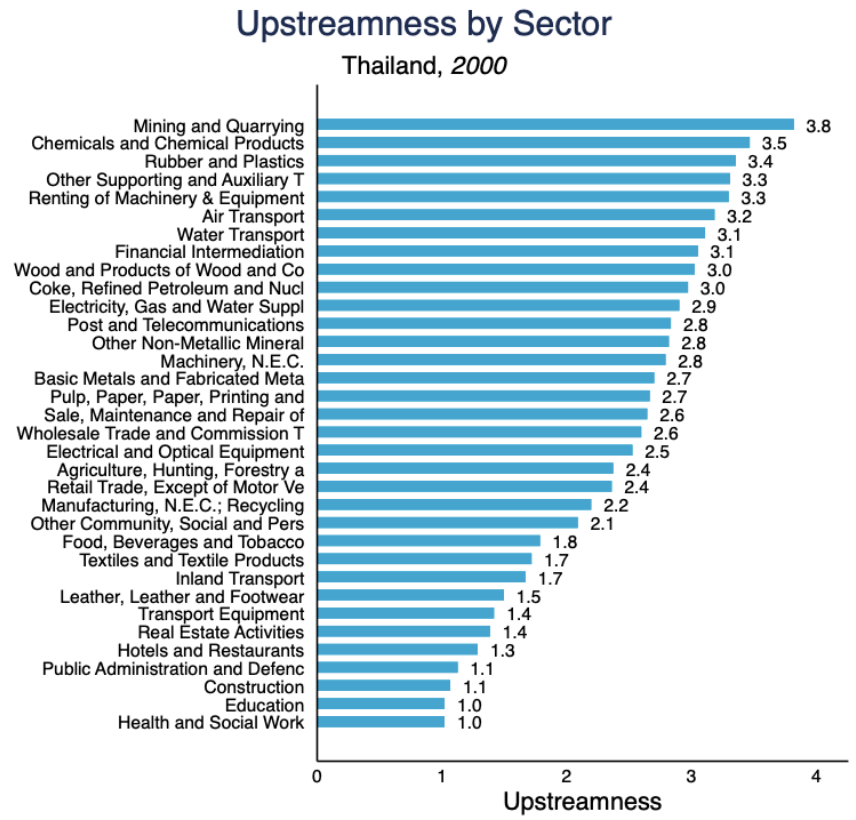
$$U = \frac{[I-A]^{-1}[I-A]^{-1}Y}{[I-A]^{-1}Y} = \frac{BBY}{BY}$$

- Provided that $\sum_{s=1}^{S}\sum_{j=1}^{J} a_{ij}^{rs} < 1$ for all j-s pairs, the numerator is equal to the $\big((i-1)\times S + r\big)$-th element of the $J\times S$ by 1 column matrix $[I-A]^{-2}Y$ or BBY.
- The upstreamness of each sector to a specific final product

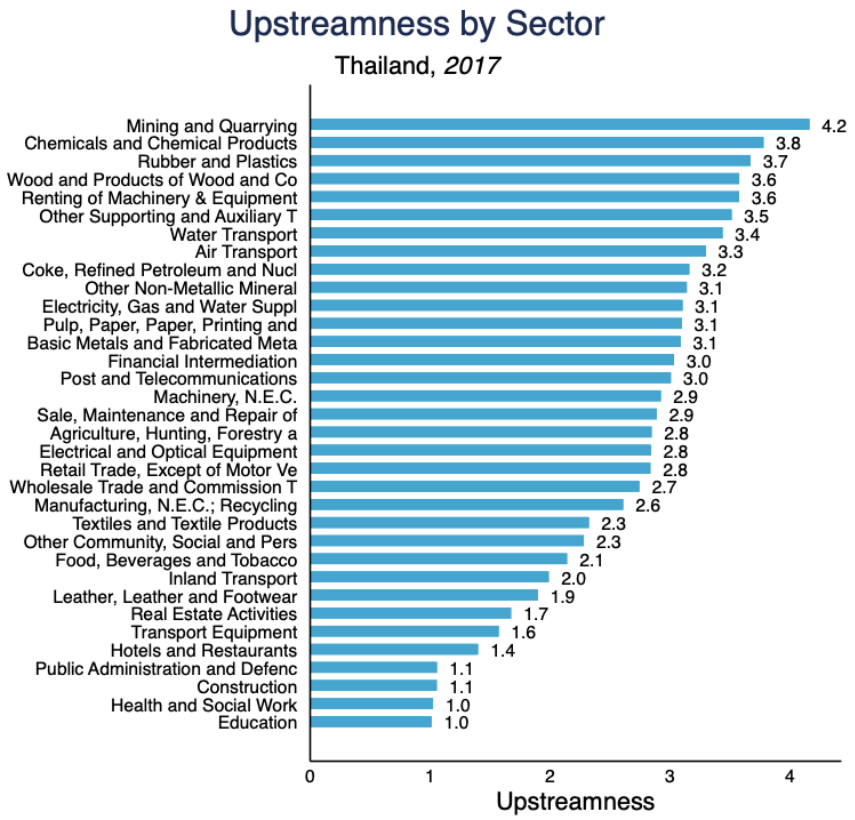
$$U = \frac{[I-A]^{-1}[I-A]^{-1}\hat{Y}}{[I-A]^{-1}\hat{Y}} = \frac{BB\hat{Y}}{B\hat{Y}} = \frac{BB}{B}$$

ADB

Many sectors in Thailand moved further away from final use in 2017 from 2000 levels.



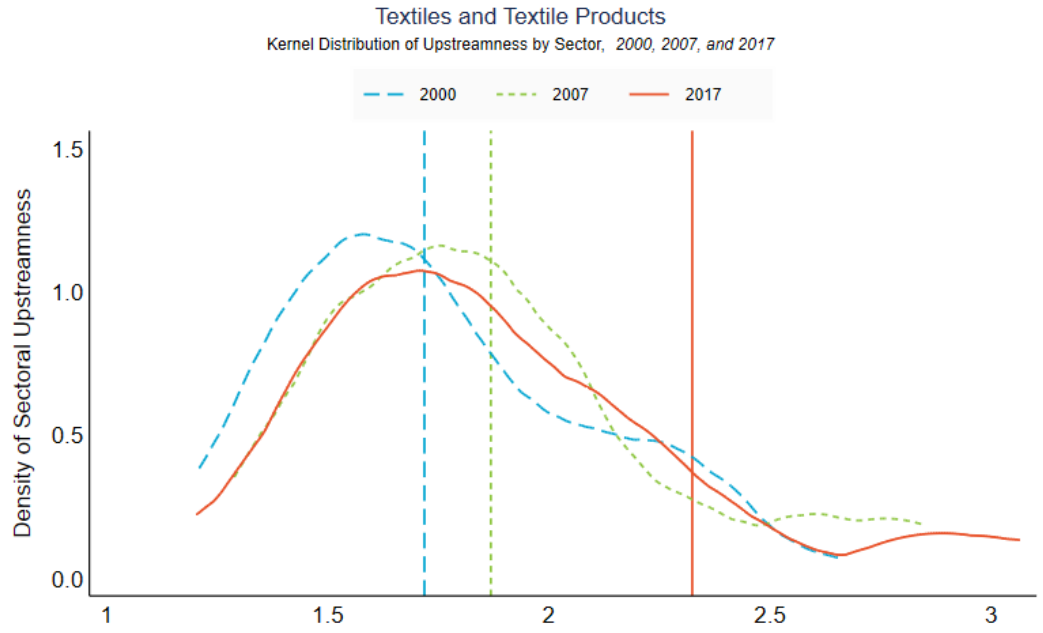
Source: Authors' estimates based on Antràs et al. (2012) using ADB-MRIO (2000, 2007-2017).



Source: Authors' estimates based on Antràs et al. (2012) using ADB-MRIO (2000, 2007-2017).

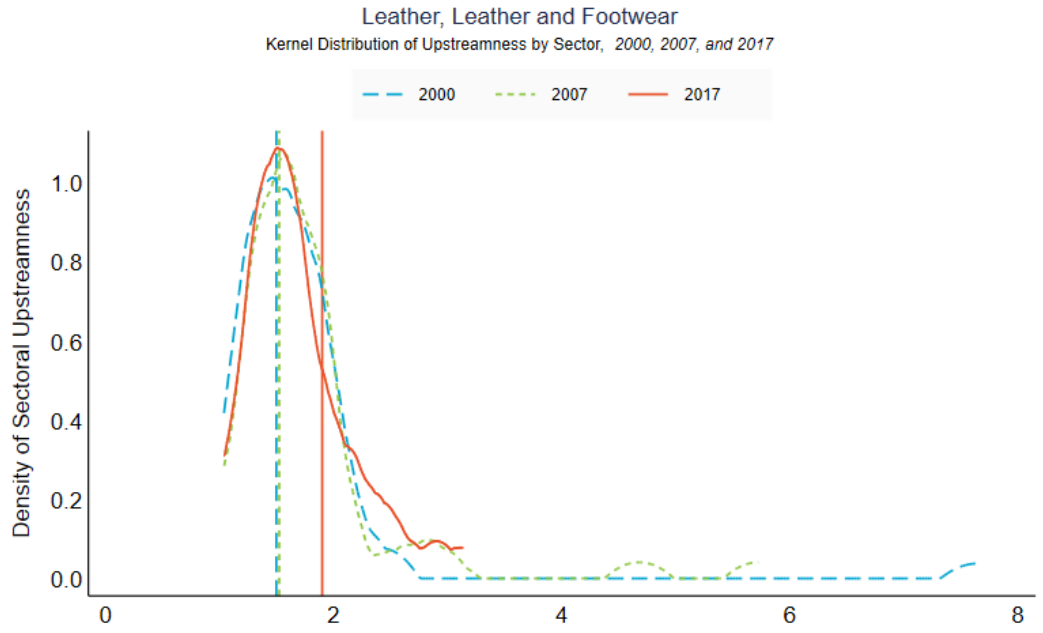
ADB

Thailand's Textiles and Leather sectors are relatively more downstream compared to average countries in those same sectors.



Mean Upstreamness, 2000: 1.8 Median Upstreamness, 2000: 1.7 Thailand Upstreamness, 2000: 1.7
Mean Upstreamness, 2007: 1.9 Median Upstreamness, 2007: 1.8 Thailand Upstreamness, 2007: 1.9
Mean Upstreamness, 2017: 1.9 Median Upstreamness, 2017: 1.8 Thailand Upstreamness, 2017: 2.3

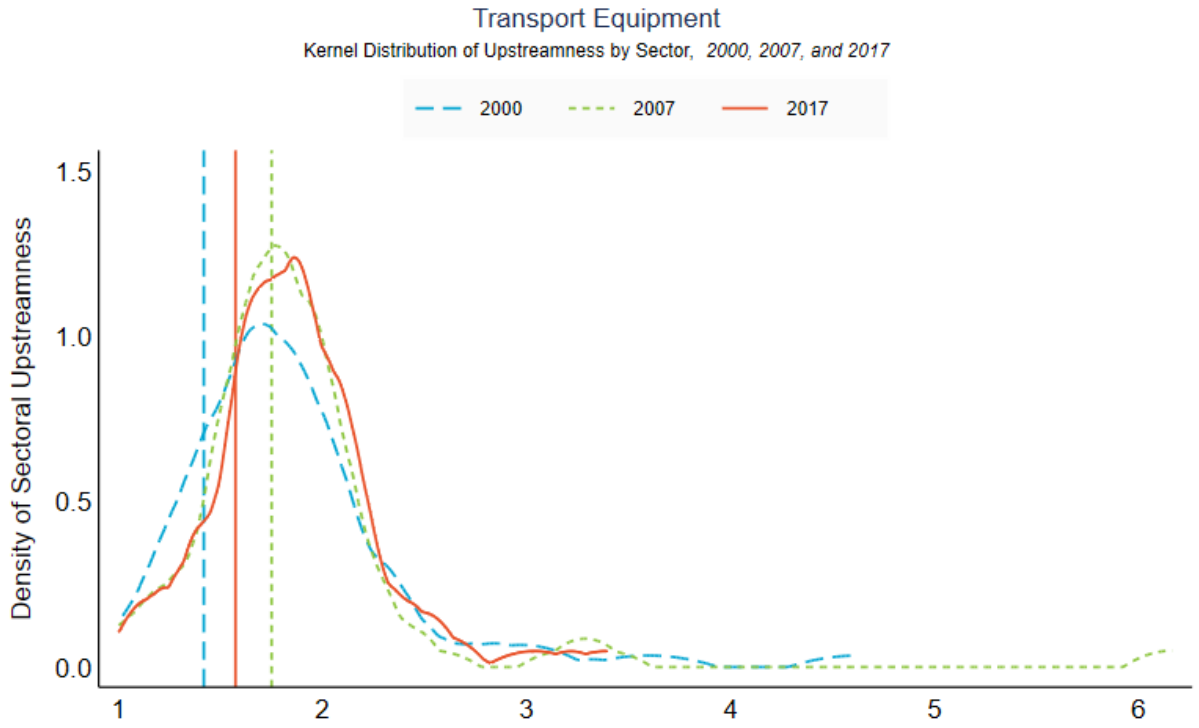
Note: Vertical lines refer to the upstreamness of Thailand.
Source: Authors' estimates based on Antràs et al. (2012) using ADB-MRIO (2000, 2007-2017).



Mean Upstreamness, 2000: 1.7 Median Upstreamness, 2000: 1.6 Thailand Upstreamness, 2000: 1.5
Mean Upstreamness, 2007: 1.8 Median Upstreamness, 2007: 1.6 Thailand Upstreamness, 2007: 1.5
Mean Upstreamness, 2017: 1.7 Median Upstreamness, 2017: 1.6 Thailand Upstreamness, 2017: 1.9

Note: Vertical lines refer to the upstreamness of Thailand.
Source: Authors' estimates based on Antràs et al. (2012) using ADB-MRIO (2000, 2007-2017).

Thailand's transport equipment sector is relatively more upstream compared to the average country involved in that sector.



Note: Vertical lines refer to the upstreamness of Thailand.
Source: Authors' estimates based on Antràs et al. (2012) using ADB-MRIO (2000, 2007-2017).

Key Takeaways

Key Takeaways

- Global value chains has gained importance over recent decades.
- Applying Leontief (1936) to the MRIO is a useful first step in analyzing country, country-sector performance within global value chains.
- GVC analysis can help us answer questions such as:
  - Who is the final consumer of value-added generated from a country-sector?
  - What is structure of the value-added source of a country-industry's final production?
  - What is the structure of exports at the country, country-sector, and bilateral level?
  - What percentage of production factors employed in a country-sector has been involved in cross country production sharing activity?
  - What percentage of final products produced by a country-sector comes from GVC activities?
  - Where is Thailand within the global production network?

ADB

Appendix

The Leontief insight

The Two-Country Production Case

- All gross output produced by Country s must be used as either an intermediate or a final product at home or abroad

$$X^s = A^{ss}X^s + Y^{ss} + A^{sr}X^r + Y^{sr} \quad r, s = 1,2$$

- X^s is the $N \times 1$ gross output vector of Country s
- Y^{sr} is the $N \times 1$ final demand vector that gives demand in Country r for final products produced in Country s
- A^{sr} is the $N \times N$ IO coefficient matrix, giving intermediate use in Country r of goods and services produced in Country s
  - Each element of A^{ij} is given by $a^{ij} = \frac{z^{ij}}{x^j}$, where z^{ij} is the output of Country i consumed by Country j

ADB

Two-Country Production Case

$$\begin{bmatrix} X^s \\ X^r \end{bmatrix} = \begin{bmatrix} A^{ss} & A^{sr} \\ A^{rs} & A^{rr} \end{bmatrix} \begin{bmatrix} X^s \\ X^r \end{bmatrix} + \begin{bmatrix} Y^{ss} + Y^{sr} \\ Y^{rs} + Y^{rr} \end{bmatrix}$$

Rearranging gives us the standard Leontief insight:

$$\begin{bmatrix} X^s \\ X^r \end{bmatrix} = \begin{bmatrix} 1 - A^{ss} & -A^{sr} \\ -A^{rs} & 1 - A^{rr} \end{bmatrix}^{-1} \begin{bmatrix} Y^{ss} + Y^{sr} \\ Y^{rs} + Y^{rr} \end{bmatrix} = \underbrace{\begin{bmatrix} B^{ss} & B^{sr} \\ B^{rs} & B^{rr} \end{bmatrix}}_{\substack{\textit{Leontief} \\ \textit{inverse}}} \begin{bmatrix} Y^s \\ Y^r \end{bmatrix}$$

The $\hat{V}B\hat{Y}$ Matrix

Intuition behind the Leontief Insight

- When $1 of gross output is produced, a first round of value added is generated. This is the direct domestic value added induced by the $1 gross output.
- To produce that gross output, intermediate inputs have to be used. The production of these intermediate inputs also generates value added. This is the second round or indirect domestic value added induced by the $1 gross output.
- Such a process to generate indirect value added continues and can be traced to additional rounds of production throughout the economy, as intermediate inputs are used to produce other intermediate inputs.

ADB

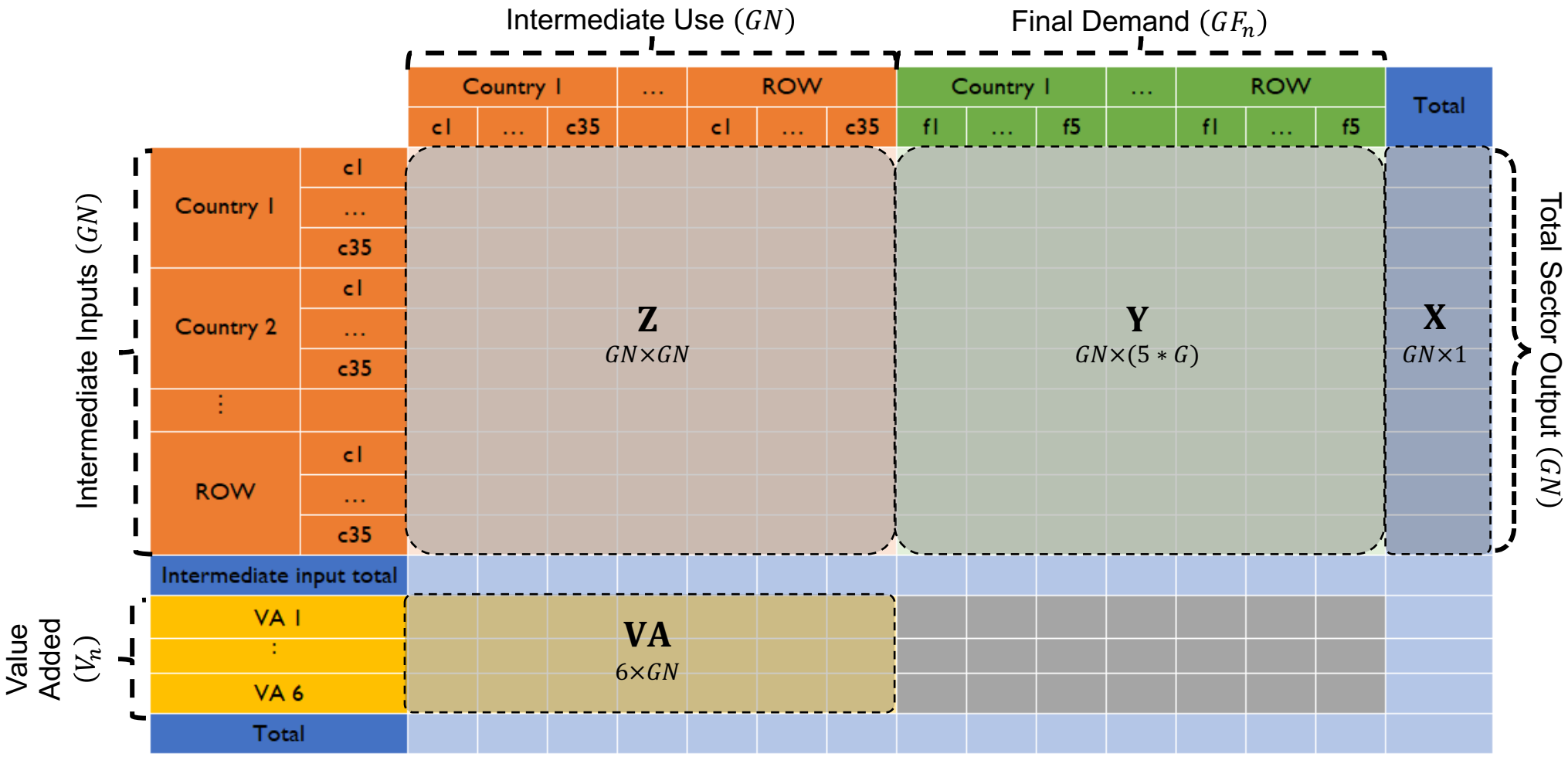
Two-Country Production Case

- The total domestic value added induced by the $1 gross output is equal to the sum of direct and all rounds of indirect domestic value added generated from the $1 of the gross output production process.

$$DVA = V + VA + VAA + VAAA + \cdots$$
$$= V(I + A + A^2 + A^3 + \cdots) = V(I - A)^{-1} = VB$$

$$VB = \begin{bmatrix} V^s & V^r \end{bmatrix} \begin{bmatrix} B^{ss} & B^{sr} \\ B^{rs} & B^{rr} \end{bmatrix} = \begin{bmatrix} V^s B^{ss} + V^r B^{rs} & V^s B^{sr} + V^r B^{rr} \end{bmatrix}$$

Partitioning the MRIO



Note:

$$V = Va\hat{X}^{-1}$$

$$A = Z\hat{X}^{-1}$$

Two-Country Production Case

- Decomposition of the country-sector level value added and final products production as a direct application of the standard Leontief decomposition:

$$\hat{V}B\hat{Y} = \begin{bmatrix} \hat{V}^s & 0 \\ 0 & \hat{V}^r \end{bmatrix} \begin{bmatrix} B^{ss} & B^{sr} \\ B^{rs} & B^{rr} \end{bmatrix} \begin{bmatrix} \hat{Y}^s & 0 \\ 0 & \hat{Y}^r \end{bmatrix}$$

$$= \begin{bmatrix} \hat{V}^s B^{ss} \hat{Y}^s & \hat{V}^s B^{sr} \hat{Y}^r \\ \hat{V}^r B^{rs} \hat{Y}^s & \hat{V}^r B^{rr} \hat{Y}^r \end{bmatrix}$$

Two-Country, Two-Sector Production Case

- Total value added coefficient (VB) matrix or the "total value added multiplier"

$$VB = \begin{bmatrix} v_1^s & v_2^s & v_1^r & v_2^r \end{bmatrix} \begin{bmatrix} b_{11}^{ss} & b_{12}^{ss} & b_{11}^{sr} & b_{12}^{sr} \\ b_{21}^{ss} & b_{22}^{ss} & b_{21}^{sr} & b_{22}^{sr} \\ b_{11}^{rs} & b_{12}^{rs} & b_{11}^{rr} & b_{12}^{rr} \\ b_{21}^{rs} & b_{22}^{rs} & b_{21}^{rr} & b_{22}^{rr} \end{bmatrix}$$

Two-Country, Two-Sector Production Case

$$\hat{V}B\hat{Y} = \begin{bmatrix} v_1^s & 0 & 0 & 0 \\ 0 & v_2^s & 0 & 0 \\ 0 & 0 & v_1^r & 0 \\ 0 & 0 & 0 & v_2^r \end{bmatrix} \begin{bmatrix} b_{11}^{ss} & b_{12}^{ss} & b_{11}^{sr} & b_{12}^{sr} \\ b_{21}^{ss} & b_{22}^{ss} & b_{21}^{sr} & b_{22}^{sr} \\ b_{11}^{rs} & b_{12}^{rs} & b_{11}^{rr} & b_{12}^{rr} \\ b_{21}^{rs} & b_{22}^{rs} & b_{21}^{rr} & b_{22}^{rr} \end{bmatrix} \begin{bmatrix} y_1^s & 0 & 0 & 0 \\ 0 & y_2^s & 0 & 0 \\ 0 & 0 & y_1^r & 0 \\ 0 & 0 & 0 & y_2^r \end{bmatrix}$$

where:

$$v_j^c = \frac{va_j^c}{x_j^c}, \qquad c = s, r \quad j = 1,2$$

Two-Country, Two-Sector Production Case

$$\hat{V}B\hat{Y} = \begin{bmatrix} v_1^s b_{11}^{ss} y_1^s & v_1^s b_{12}^{ss} y_2^s & v_1^s b_{11}^{sr} y_1^r & v_1^s b_{12}^{sr} y_2^r \\ v_2^s b_{21}^{ss} y_1^s & v_2^s b_{22}^{ss} y_2^s & v_2^s b_{21}^{sr} y_1^r & v_2^s b_{22}^{sr} y_2^r \\ v_1^r b_{11}^{rs} y_1^s & v_1^r b_{12}^{rs} y_2^s & v_1^r b_{11}^{rr} y_1^r & v_1^r b_{12}^{rr} y_2^r \\ v_2^r b_{21}^{rs} y_1^s & v_2^r b_{22}^{rs} y_2^s & v_2^r b_{21}^{rr} y_1^r & v_2^r b_{22}^{rr} y_2^r \end{bmatrix}$$

- This matrix gives the estimate of sector and country sources of value added in each country's final goods production.
- Each element in the matrix represents the value added from a source sector of a source country directly or indirectly used in the production of final goods (absorbed in both the domestic and foreign markets) in the source country.

ADB

Interpreting the $\hat{V}B\hat{Y}$ Matrix: Row-wise

- Looking at the matrix along a row yields the distribution of value added created from one country-sector used across all countries-sectors.

$$\hat{V}B\hat{Y} = \begin{bmatrix} v_1^s b_{11}^{ss} y_1^s & v_1^s b_{12}^{ss} y_2^s & v_1^s b_{11}^{sr} y_1^r & v_1^s b_{12}^{sr} y_2^r \\ v_2^s b_{21}^{ss} y_1^s & v_2^s b_{22}^{ss} y_2^s & v_2^s b_{21}^{sr} y_1^r & v_2^s b_{22}^{sr} y_2^r \\ v_1^r b_{11}^{rs} y_1^s & v_1^r b_{12}^{rs} y_2^s & v_1^r b_{11}^{rr} y_1^r & v_1^r b_{12}^{rr} y_2^r \\ v_2^r b_{21}^{rs} y_1^s & v_2^r b_{22}^{rs} y_2^s & v_2^r b_{21}^{rr} y_1^r & v_2^r b_{22}^{rr} y_2^r \end{bmatrix}$$

- $v_1^s b_{11}^{ss}(y_1^{ss} + y_1^{sr})$ is the value added from sector 1 of Country s to produce the final products of sector 1 for domestic sales and exports.
- $v_1^s b_{12}^{ss}(y_2^{ss} + y_2^{sr})$ is the value added from sector 1 of Country s from production of intermediates used as inputs to produce final products of sector 2.
- $v_1^s b_{11}^{ss}(y_1^{rs} + y_1^{rr})$ is the value added from sector 1 of Country s from the production of intermediate inputs used to produce final products in sector 1 of in Country r.
- $v_1^s b_{12}^{ss}(y_2^{rs} + y_2^{rr})$ is the value added from sector 1 of Country s from the production of intermediate inputs used to produce final products in sector 2 of in Country r.

ADB

Interpreting the $\hat{V}B\hat{Y}$ Matrix: Row-wise

- Looking at the matrix along a row yields the distribution of value added created from one country-sector used across all countries-sectors.

$$\hat{V}B\hat{Y} = \begin{bmatrix} v_1^s b_{11}^{ss} y_1^s & v_1^s b_{12}^{ss} y_2^s & v_1^s b_{11}^{sr} y_1^r & v_1^s b_{12}^{sr} y_2^r \\ v_2^s b_{21}^{ss} y_1^s & v_2^s b_{22}^{ss} y_2^s & v_2^s b_{21}^{sr} y_1^r & v_2^s b_{22}^{sr} y_2^r \\ v_1^r b_{11}^{rs} y_1^s & v_1^r b_{12}^{rs} y_2^s & v_1^r b_{11}^{rr} y_1^r & v_1^r b_{12}^{rr} y_2^r \\ v_2^r b_{21}^{rs} y_1^s & v_2^r b_{22}^{rs} y_2^s & v_2^r b_{21}^{rr} y_1^r & v_2^r b_{22}^{rr} y_2^r \end{bmatrix}$$

- Summing up the first row of the matrix gives the total value added created by production factors employed in sector 1 of Country s. [GDP by industry of sector 1 in Country s]

$$va_1^s \; or GDP_1^s = v_1^s x_1^s = v_1^s (b_{11}^{ss} y_1^s + b_{12}^{ss} y_2^s + b_{11}^{sr} y_1^r + b_{12}^{sr} y_2^s)$$
$$= [v_1^s b_{11}^{ss} (y_1^{ss} + y_1^{sr}) + v_1^s b_{12}^{ss} (y_2^{ss} + y_2^{sr})] + [v_1^s b_{11}^{sr} (y_1^{rs} + y_1^{rr}) + v_1^s b_{12}^{sr} (y_2^{rs} + y_2^{rr})]$$

ADB

Interpreting the $\hat{V}B\hat{Y}$ Matrix: Column-wise

- Looking at the matrix along a row yields the distribution of value added created from one country-sector used across all countries-sectors.

$$\hat{V}B\hat{Y} = \begin{bmatrix} v_1^s b_{11}^{ss} y_1^s & v_1^s b_{12}^{ss} y_2^s & v_1^s b_{11}^{sr} y_1^r & v_1^s b_{12}^{sr} y_2^r \\ v_2^s b_{21}^{ss} y_1^s & v_2^s b_{22}^{ss} y_2^s & v_2^s b_{21}^{sr} y_1^r & v_2^s b_{22}^{sr} y_2^r \\ v_1^r b_{11}^{rs} y_1^s & v_1^r b_{12}^{rs} y_2^s & v_1^r b_{11}^{rr} y_1^r & v_1^r b_{12}^{rr} y_2^r \\ v_2^r b_{21}^{rs} y_1^s & v_2^r b_{22}^{rs} y_2^s & v_2^r b_{21}^{rr} y_1^r & v_2^r b_{22}^{rr} y_2^r \end{bmatrix}$$

- $v_1^s b_{11}^{ss}(y_1^{ss} + y_1^{sr})$ is the value added created from sector 1 in Country s in its production of intermediate inputs used by sector 1 in Country s to produce its final products.
- $v_2^s b_{21}^{ss}(y_1^{ss} + y_1^{sr})$ is the value added created from sector 2 in Country s in its production of intermediate inputs used by sector 1 in Country s to produce its final products.
- $v_1^r b_{11}^{rs}(y_1^{ss} + y_1^{sr})$ is the value added created from sector 1 in Country r in its production of intermediate inputs used by sector 1 in Country s to produce its final products.
- $v_2^r b_{21}^{rs}(y_1^{ss} + y_1^{sr})$ is the value added created from sector 2 in Country r in its production of intermediate inputs used by sector 1 in Country s to produce its final products.

ADB

Interpreting the $\hat{V}B\hat{Y}$ Matrix: Column-wise

- Looking at the matrix along a row yields the distribution of value added created from one country-sector used across all countries-sectors.

$$\hat{V}B\hat{Y} = \begin{bmatrix} v_1^s b_{11}^{ss} y_1^s & v_1^s b_{12}^{ss} y_2^s & v_1^s b_{11}^{sr} y_1^r & v_1^s b_{12}^{sr} y_2^r \\ v_2^s b_{21}^{ss} y_1^s & v_2^s b_{22}^{ss} y_2^s & v_2^s b_{21}^{sr} y_1^r & v_2^s b_{22}^{sr} y_2^r \\ v_1^r b_{11}^{rs} y_1^s & v_1^r b_{12}^{rs} y_2^s & v_1^r b_{11}^{rr} y_1^r & v_1^r b_{12}^{rr} y_2^r \\ v_2^r b_{21}^{rs} y_1^s & v_2^r b_{22}^{rs} y_2^s & v_2^r b_{21}^{rr} y_1^r & v_2^r b_{22}^{rr} y_2^r \end{bmatrix}$$

- Summing up the first column of the matrix gives the value of final products of sector in Country s

$$v_1^s b_{11}^{ss}(y_1^{ss} + y_1^{sr}) + v_2^s b_{21}^{ss}(y_1^{ss} + y_1^{sr}) + v_1^r b_{11}^{rs}(y_1^{ss} + y_1^{sr}) + v_2^r b_{21}^{rs}(y_1^{ss} + y_1^{sr})$$
$$= (v_1^s b_{11}^{ss} + v_2^s b_{21}^{ss} + v_1^r b_{11}^{rs} + v_2^r b_{21}^{rs} + v_2^r b_{21}^{rs}) y_1^s = y_1^s$$

Decomposition of Gross Trade into Value-Added Terms

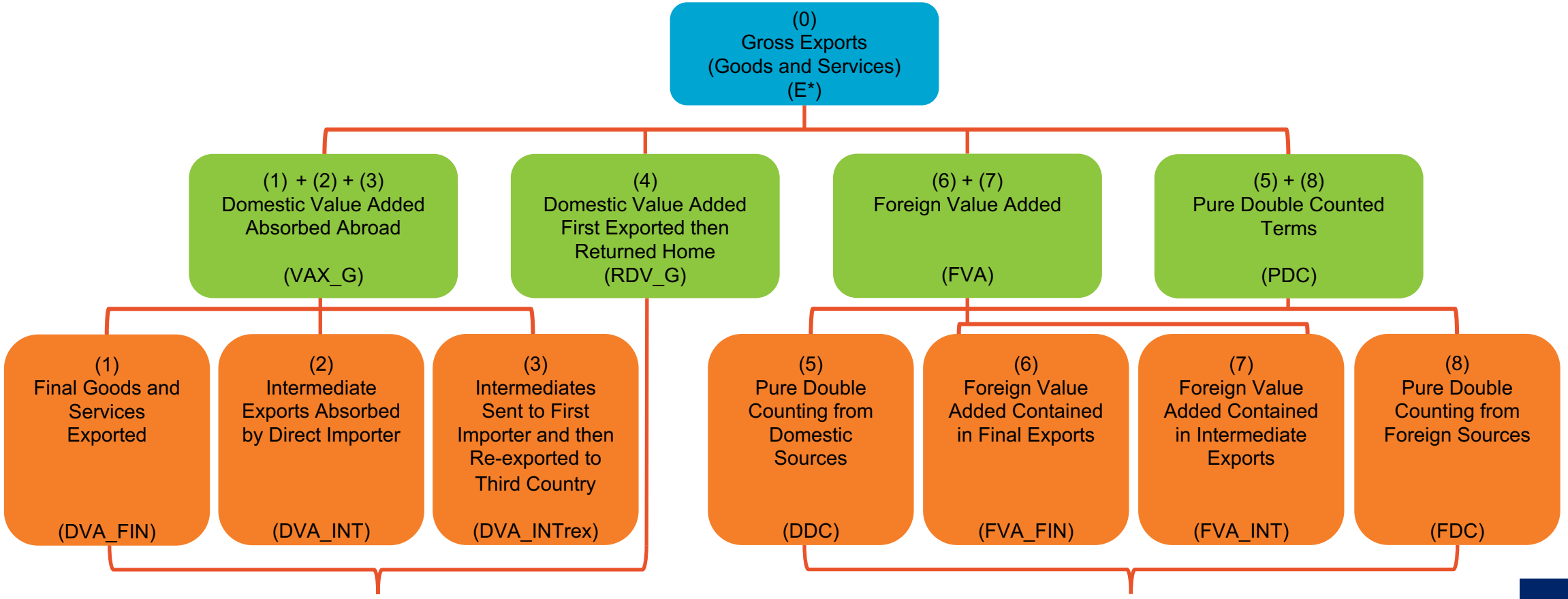
Koopman, R., Wang, Z., Wei, S.J. (2014). Tracing value-added and double counting in gross exports. *American Economic Review, 104*(2), 459–494. doi: 10.1257/aer.104.2.459

Wang, Z., Wei, S.J., & Zhu, K. (2018). *Quantifying international production sharing at the bilateral and sectoral levels* (NBER Working Paper 19677). Cambridge, MA: National Bureau of Economic Research. Retrieved from the National Bureau of Economic Research: https://www.nber.org/papers/w19677

WWZ (2018)

- Builds on Koopman, Wang, and Wei (2014)
  - Decomposes gross trade at the sector, bilateral, or bilateral-sector level.
  - Distinguishes between backward and forward linkages, allowing decomposition at a disaggregated level
  - Distinguishes two types of “trade in value-added” measures and two types of domestic value-added embedded in gross exports based on forward and backward industrial linkages at the country-sector or bilateral-sector levels

Gross Trade Accounting: Conceptual Framework



Note: E* can be at country-sector, country aggregate, bilateral-sector, or bilateral aggregate. Both VAX_G and RDV_G are based on backward industrial linkages.

Decomposing Intermediate and Gross Trade Flows

- The gross exports of Country s to Country r, E^{sr} can be completely decomposed into the sum of 16 detailed terms in 8 major categories.

$$E^{sr} = \underbrace{(V^s B^{ss})^T \# Y^{sr}}_{(1)DVA\_FIN}$$

$$+ \underbrace{(V^s L^{ss})^T \# (A^{sr} B^{rr} Y^{rr})^T}_{(2)DVA\_INT}$$

$$+ \underbrace{(V^s L^{ss})^T \# \left[A^{sr} \sum_{t \neq s,r}^{G} B^{rt} Y^{tt} + A^{sr} B^{rr} \sum_{t \neq s,r}^{G} Y^{rt} + A^{sr} \sum_{t \neq s,r}^{G} B^{rt} \sum_{u \neq s,t}^{G} Y^{tu} \right]}_{(3)DVA\_INTrex}$$

$$+ \underbrace{(V^s L^{ss})^T \# \left[A^{sr} B^{rr} Y^{rs} + A^{sr} \sum_{t \neq s,r}^{G} B^{rt} Y^{ts} + A^{sr} B^{rs} Y^{ss} \right]}_{(4)RDV\_G}$$

$$+ \underbrace{\left[(V^s L^{ss})^T \# \left(A^{sr} B^{rs} \sum_{t \neq s}^{G} Y^{st} \right) + \left(V^s L^{ss} \sum_{t \neq s}^{G} A^{st} B^{ts} \right)^T \# (A^{sr} X^r) \right]}_{(5)DDC}$$

$$+ \underbrace{\left[(V^r B^{rs})^T \# Y^{sr} + \left(\sum_{t \neq s,r}^{G} V^t B^{ts} \right)^T \# Y^{sr} \right]}_{(6)FVA\_FIN}$$

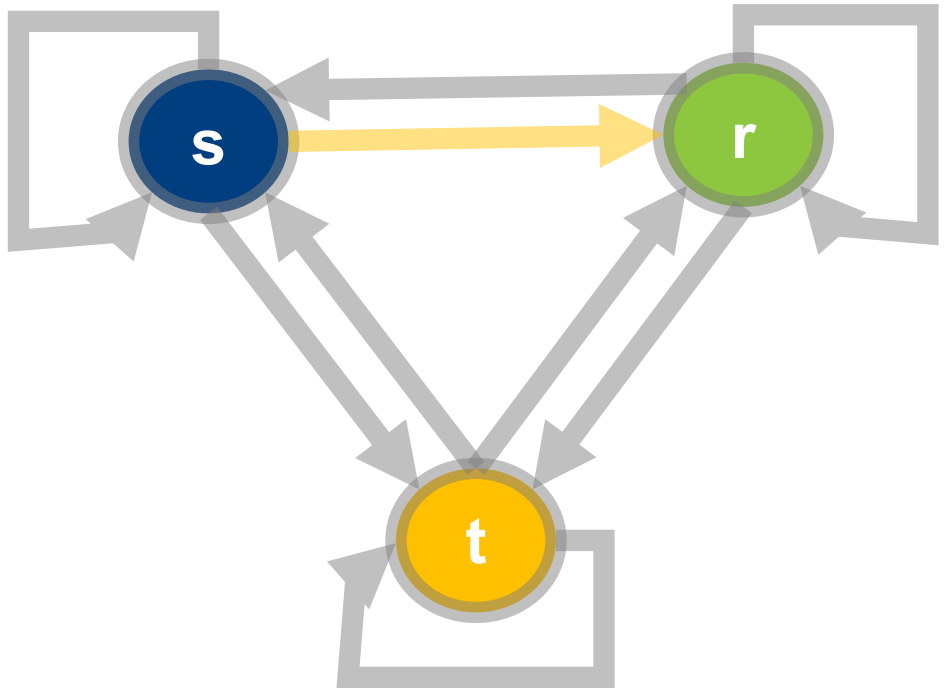
$$+ \underbrace{\left[(V^r B^{rs})^T \# (A^{sr} L^{rr} Y^{rr}) + \left(\sum_{t \neq s,r}^{G} V^t B^{ts} \right)^T \# (A^{sr} L^{rr} Y^{rr}) \right]}_{(7)FVA\_INT}$$

$$+ \underbrace{\left[(V^r B^{rs})^T \# \left(A^{sr} L^{rr} E^{r^*} \right) + \left(\sum_{t \neq s,r}^{G} V^t B^{ts} \right)^T \# \left(A^{sr} L^{rr} E^{r^*} \right) \right]}_{(7)FDC}$$

(1) DVA_FIN

$$(V^s B^{ss})^T \# Y^{sr}$$

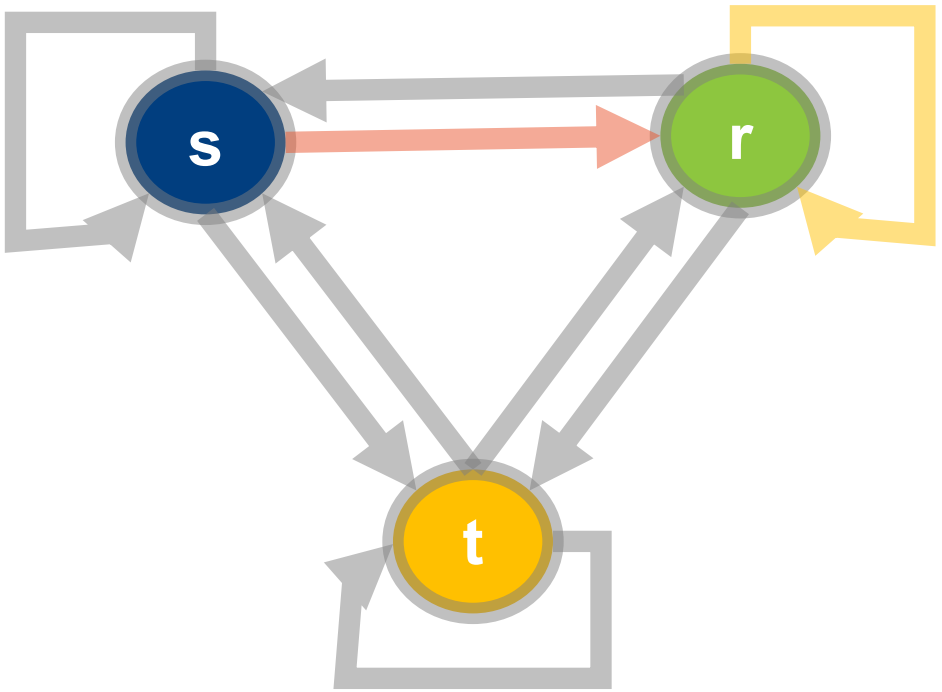
- Domestic value added embodied in Country s's final goods exports.



(2) DVA_INT

$$(V^s L^{ss})^T \#(A^{sr} B^{rr} Y^{rr})^T$$

- Domestic value added in Country s's intermediate exports used by direct importer (r) to produce local final goods consumed in r.



(3) DVA_INTrex

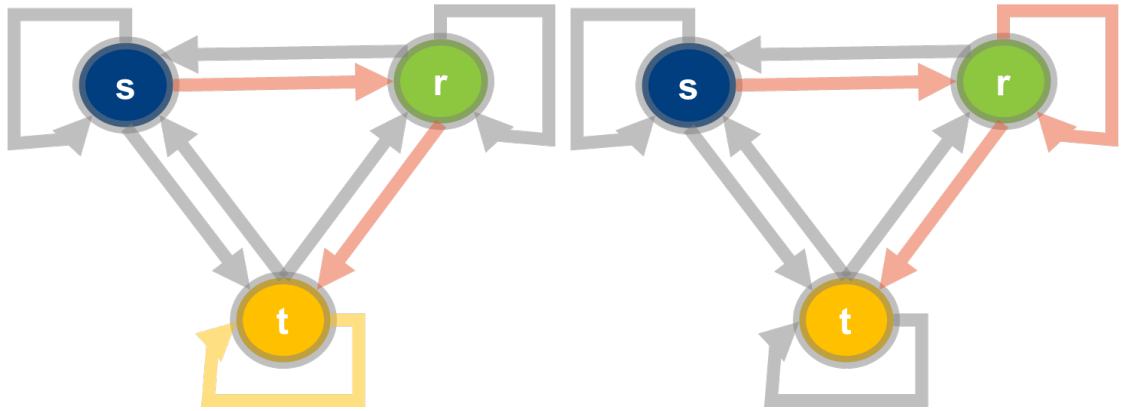
$$(V^s L^{ss})^T \# \left[A^{sr} \sum_{t \neq s,r}^{G} B^{rt} Y^{tt} + A^{sr} B^{rr} \sum_{t \neq s,r}^{G} Y^{rt} + A^{sr} \sum_{t \neq s,r}^{G} B^{rt} \sum_{u \neq s,t}^{G} Y^{tu} \right]$$

- Domestic value added in intermediate exports used by direct importer (r) to produce exports ultimately consumed by other countries except s

  (3a) $(V^s L^{ss})^T \# \left(A^{sr} \sum_{t \neq s,r}^{G} B^{rt} Y^{tt} \right)$: DVA in to Country s's intermediate exports that are used by Country r to produce intermediates that it re-exports to third Country t for production of local final goods;

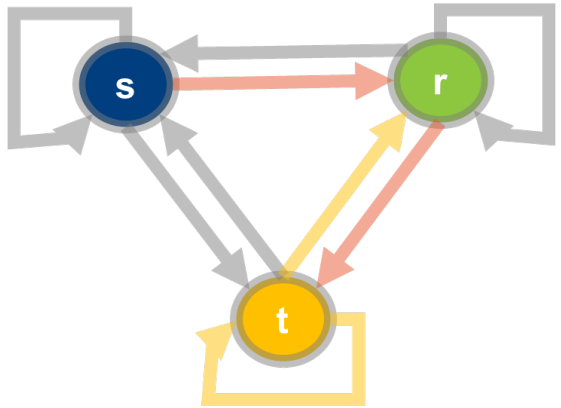
  (3b) $(V^s L^{ss})^T \# \left(A^{sr} B^{rr} \sum_{t \neq s,r}^{G} Y^{rt} \right)$: DVA in to Country s's intermediate exports used by Country r to produce final goods that it re-exports to third Country t;

  (3c) $(V^s L^{ss})^T \# \left(A^{sr} \sum_{t \neq s,r}^{G} B^{rt} \sum_{u \neq s,t}^{G} Y^{tu} \right)$: DVA in Country s's intermediate exports used by Country r to produce intermediates that it re-exports to third Country t for production of final goods exports that are shipped to other countries (including Country r) except Country s.



(3a)

(3b)



(3c)

VAX_G = DVA_FIN + DVA_INT + DVA_INTrex

- VAX_G measures all domestic value added embodied in Country s's gross exports to Country *r* which are ultimately absorbed abroad
- VAX_G are value-added exports associated with gross export flows based on backward industrial linkages.

(4) RDV_G

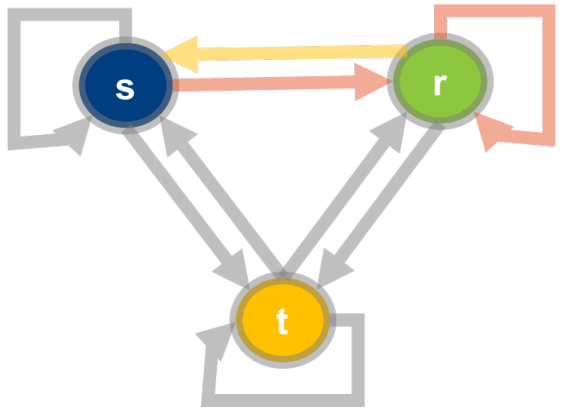
$$(V^s L^{ss})^T \# \left[A^{sr} B^{rr} Y^{rs} + A^{sr} \sum_{t \neq s,r}^{G} B^{rt} Y^{ts} + A^{sr} B^{rs} Y^{ss} \right]$$

- Domestic value added in intermediate exports that are returned to Country s and consumed at home.

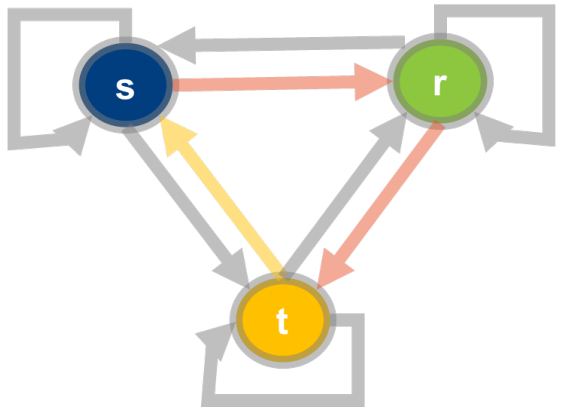
  (4a) $(V^s L^{ss})^T \# (A^{sr} B^{rr} Y^{rs})$: DVA that returns home to Country s via its final imports from the direct importer (r);

  (4b) $(V^s L^{ss})^T \# \left(A^{sr} \sum_{t \neq s,r}^{G} B^{rt} Y^{ts}\right)$: DVA that returns home to Country s via final imports from third countries;

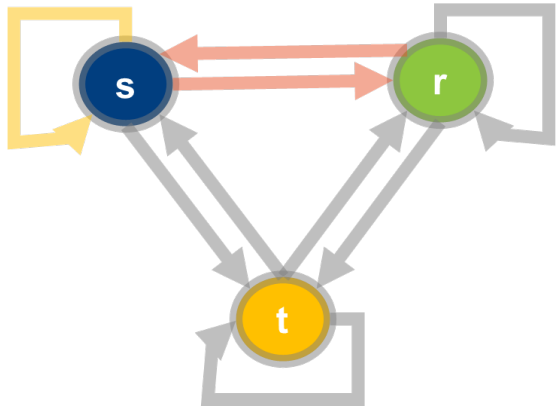
  (4c) $(V^s L^{ss})^T \# (A^{sr} B^{rs} Y^{ss})$: DVA that returns to Country s via intermediate imports from the direct importer (Country r) and used to produce domestically used final products.



(4a)



(4b)



(4c)

DVA_G = DVA_FIN + DVA_INT + DVA_INTrex + RDV_G

- DVA_G measures domestic value added (GDP by industry) embodied in Country s's sector level gross exports to Country r, which include value added created from all sectors in Country s.
- These DVA terms represent different types of cross country production sharing arrangements and can be used to gauge the role and position of a country in various global value chains.

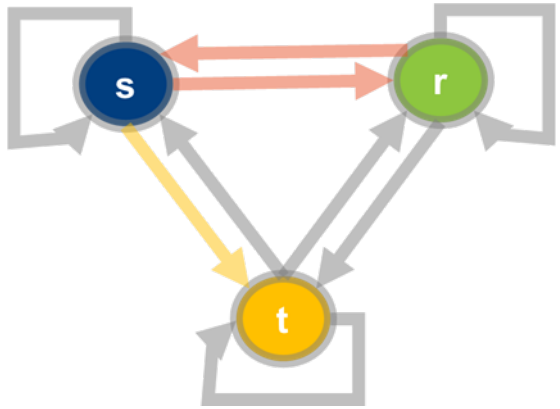
(5) DDC

$$\left[(V^s L^{ss})^T \# \left(A^{sr} B^{rs} \sum_{t \neq s}^{G} Y^{st}\right) + \left(V^s L^{ss} \sum_{t \neq s}^{G} A^{st} B^{ts}\right)^T \# (A^{sr} X^r)\right]$$

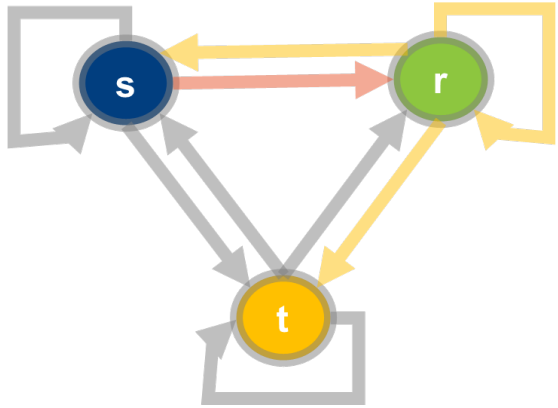
- Domestic double counted terms

  (5a)$(V^s L^{ss})^T \# (A^{sr} B^{rs} \sum_{t \neq s}^{G} Y^{st})$: DVA embodied in Country s's intermediate exports to Country r but return home as its intermediate imports, and used for production of its final exports, which are part of DVA in Country s's final exports and are already counted once in DVA_FIN;

  (5b) $(V^s L^{ss} \sum_{t \neq s}^{G} A^{st} B^{ts})^T \# (A^{sr} X^r)$: DVA in Country s's intermediate exports to Country r that returns home as intermediate imports and used for production of its immediate exports. It is also a domestic double counted portion caused by the back and forth intermediate trade to product intermediate exports in Country s (repeat counting of Country s's intermediate goods exports).



(5a)



(5b)

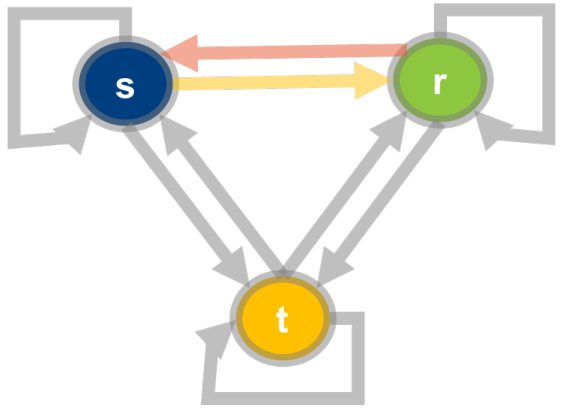
(6) FVA_FIN

$$\left[(V^r B^{rs})^T \#Y^{sr} + \left(\sum_{t \neq s,r}^{G} V^t B^{ts}\right)^T \#Y^{sr}\right]$$

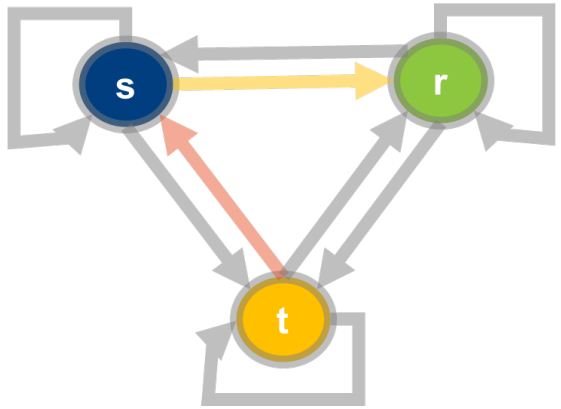
- Foreign value added embodied in final exports

  (6a)$(V^r B^{rs})^T \#Y^{sr}$: Foreign value added (FVA) from the importer (r) embodied in Country s's final exports;

  (6b)$\left(\sum_{t \neq s,r}^{G} V^t B^{ts}\right)^T \#Y^{sr}$: Foreign value added from other countries (t) embodied in Country s's final exports.



(6a)



(6b)

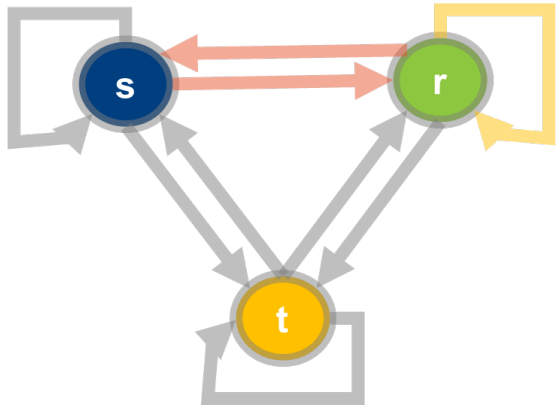
(7) FVA_INT

$$\left[(V^r B^{rs})^T \#(A^{sr} L^{rr} Y^{rr}) + \left(\sum_{t \neq s,r}^{G} V^t B^{ts}\right)^T \#(A^{sr} L^{rr} Y^{rr})\right]$$

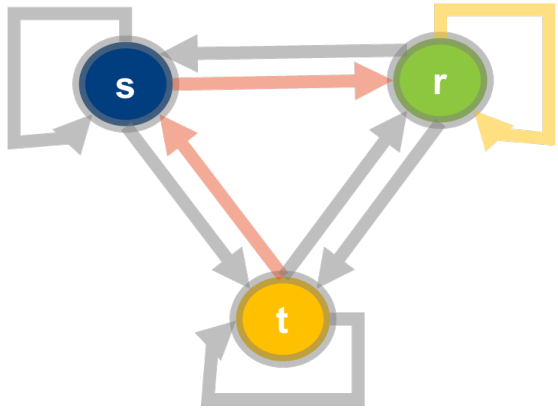
- Foreign value added embodied in intermediate exports

  (7a)$(V^r B^{rs})^T \#(A^{sr} L^{rr} Y^{rr})$: Foreign value added from the importer (r) embodied in Country s's intermediate exports, which are then used by Country r to produce its domestic final goods;

  (7b)$\left(\sum_{t \neq s,r}^{G} V^t B^{ts}\right)^T \#(A^{sr} L^{rr} Y^{rr})$: Foreign value added from third Country t embodied in Country s's intermediate exports, which are then used by Country r to produce its local final goods.



(7a)



(7b)

FVA = FVA_FIN + FVA_INT

- FVA is the total foreign value added embodied in Country s's sector level gross exports to Country r.

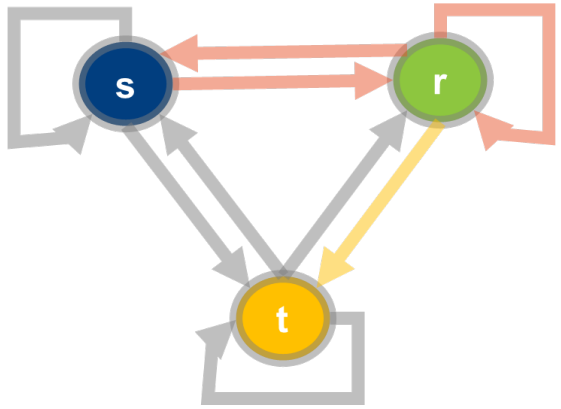
(8) FDC

$$\left[(V^r B^{rs})^T \#\left(A^{sr} L^{rr} E^{r^*}\right) + \left(\sum_{t \neq s,r}^{G} V^t B^{ts}\right)^T \#\left(A^{sr} L^{rr} E^{r^*}\right)\right]$$

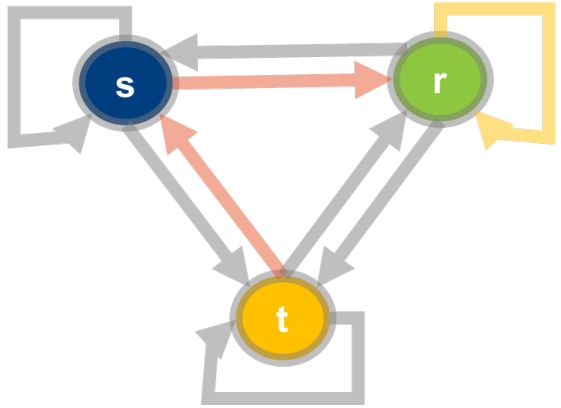
- Foreign double counted: double counted terms in Country s's gross exports originating from foreign countries.

  (8a) $(V^r B^{rs})^T \#\left(A^{sr} L^{rr} E^{r^*}\right)$: Foreign value added from the importer (r) embodied in Country s's intermediate exports to Country r, which Country r then uses to produce its final and intermediate exports. This is a pure double counted term of r's value added in s's exports;

  (8b) $\left(\sum_{t \neq s,r}^{G} V^t B^{ts}\right)^T \#\left(A^{sr} L^{rr} E^{r^*}\right)$: Foreign value added from third Country t embodied in Country s's intermediate exports to Country r, which are then used by Country r to produce its exports to the world.



(8a)



(8b)

PDC = DDC + FDC

- PDC measures pure double counted terms in Country s's gross exports but originating from home country and foreign countries.
- DDC and FDC only occur when there are back and forth intermediate trade flows that cross national borders at least 3 times but through different routes:
  - as intermediate exports from the home country,
  - returned from the direct importer or re-exported by these direct importing countries to third countries as intermediate exports, and
  - exported by a third country again to other countries including the home country as intermediate or final products.

VS = DDC + FVA_FIN + FVA_INT + FDC

- The sum of DDC, FVA_FIN, FVA_INT, and FDC can be seen as an extension of the vertical specialization (VS) measure proposed by Hummels, Ishii, and Yi (2001) in a multi-country setting with more than one country engaging in intermediate goods trade.
- These different components within the total VS also represent different types of cross-country production sharing arrangements and are useful to study the upstream value-added structure of a country's gross exports in various global value chains.

Participation in Global Value Chains

Wang, Z. Wei, S.J., Yu, X. Zhu, K. (2017). *Measures in participation in global value chains and global business cycles (*NBER Working Paper 23222). Cambridge, MA: National Bureau of Economic Research. Retrieved from the National Bureau of Economic Research: https://www.nber.org/ papers/w23222

Decomposition of Production Activities

- We can classify production activities based on factor content embodied in various products and whether these embedded factor content cross national borders for production or not.
  - Non-GVC:
    - Pure domestic demand – without involving trade
    - Traditional international trade - without involving trade in intermediate goods
  - GVC: value-added creation is only classified as GVC activities when embodied factor content crosses national border for production purposes.
    - Simple global value chain activities
    - Complex global value chain activities

Decomposition of Value Added and Final Products Production

- Starting with the Leontief Insight (1936) and converting gross outputs (and gross exports) to exogenous variables in the gross trade accounting framework, we get:

$$\hat{V}B\hat{Y} = \hat{V}L\hat{Y}^D + \hat{V}L\hat{Y}^F + \hat{V}LA^FB\hat{Y}$$
$$= \hat{V}L\hat{Y}^D + \hat{V}L\hat{Y}^F + \hat{V}LA^FL\hat{Y}^D + \hat{V}LA^F(B\hat{Y} - L\hat{Y}^D)$$

where:

$\hat{V}$ is a diagonal matrix of value added coefficients

$\hat{Y}^D$ is a diagonal matrix with the elements of the Y^D vector along the diagonal

$\hat{Y}^F$ is a diagonal matrix with the elements of the Y^F vector along the diagonal

$B = (I - A)^{-1}$ is the global Leontief inverse matrix

$L = (I - A^D)^{-1}$ is the local Leontief inverse (a GN by GN diagonal block matrix)

ADB

Decomposition of Value Added and Final Products Production

- Each element in the $\hat{V}B\hat{Y}$ matrix represents the value added from a source country-sector directly or indirectly used in the production of final goods and services in a particular country-sector.
- The element of row (s,i) and the column (r,j) in the matrix, $v_i^s b_{ij}^{sr} y_j^r$, is the total value added (direct or indirect) of sector i in country s embodied in the final products produced by sector j of country r.
- Looking at the matrix along a row yields the distribution of value added created from one country-sector that is absorbed by final goods production in all country-sectors (forward linkage perspective).
- Looking at the matrix along a column yields the contribution of value added from all source country-sectors pairs that is embodied in the final goods and services produced by a particular country-sector (backward linkage perspective).

Decomposition of the $\hat{V}B\hat{Y}$ Matrix into Production Activities

$$\hat{V}B\hat{Y} = \underbrace{\hat{V}L\hat{Y}^D}_{(1)} + \underbrace{\hat{V}L\hat{Y}^F}_{(2)} + \underbrace{\hat{V}LA^FB\hat{Y}}_{(3)} = \hat{V}L\hat{Y}^D + \hat{V}L\hat{Y}^F + \underbrace{\hat{V}LA^FL\hat{Y}^D}_{(3a)} + \underbrace{\hat{V}LA^F(B\hat{Y} - L\hat{Y}^D)}_{(3b)}$$

(1) $\hat{V}L\hat{Y}^D$: value added that is domestically produced and consumed. This value added does not involve cross border trade.

(2) $\hat{V}L\hat{Y}^F$: value added that is embodied in final product exports. This embodied domestic factor content crosses national borders for consumption only. It is similar to “traditional” trade.

(3) $\hat{V}LA^FB\hat{Y}$: value added that is embodied in exports/imports of intermediate goods and services. Because it is used in production activities outside the source country, it is part of the cross-country production sharing activities.

(3a) $\hat{V}LA^FL\hat{Y}^D$: simple cross country production sharing activities. Domestic or/and foreign value-added cross national border for production only once. Value added embodied in intermediate exports/imports that is used by a direct importing country to produce products that are absorbed in the country. There is no indirect exports via third countries or re-exports/re-imports of the source countries’ factor contents.

(3b) $\hat{V}LA^F(B\hat{Y} - L\hat{Y}^D)$: complex cross country production sharing activities. Domestic or/and foreign value added embodied in intermediate exports/imports that is used by partner country to produce exports (intermediate or final) for other countries. The factor contents cross border at least twice.

ADB

Decomposition of the $\hat{V}B\hat{Y}$ Matrix into Production Activities

$$\hat{V}B\hat{Y} = \underbrace{\hat{V}L\hat{Y}^D}_{(1)} + \underbrace{\hat{V}L\hat{Y}^F}_{(2)} + \underbrace{\hat{V}LA^FB\hat{Y}}_{(3)} = \hat{V}L\hat{Y}^D + \hat{V}L\hat{Y}^F + \underbrace{\hat{V}LA^FL\hat{Y}^D}_{(3a)} + \underbrace{\hat{V}LA^F(B\hat{Y} - L\hat{Y}^D)}_{(3b)}$$

Summing along the **row direction**, we can decompose value-added generated from each country-sector (GDP by industry) in terms of where it goes:

$$Va' = \hat{V}BY = \underset{(1)V\_D}{\hat{V}LY^D} + \underset{(2)V\_RT}{\hat{V}LY^F} + \underset{(3a)Y\_GVC\_S}{\hat{V}LA^FLY^D} + \underbrace{\hat{V}LA^F(BY - LY^D)}_{(3b)Y\_GVC\_C}$$

Summing along the **column direction**, we can decompose country-sector final goods production in terms of where the value added comes from:

$$Y' = VB\hat{Y} = \underset{(1)Y\_D}{VL\hat{Y}^D} + \underset{(2)Y\_RT}{VL\hat{Y}^F} + \underset{(3a)Y\_GVC\_S}{VLA^FL\hat{Y}^D} + \underbrace{VLA^F(B\hat{Y} - L\hat{Y}^D)}_{(3b)Y\_GVC\_C}$$

ADB

Decomposition of the $\hat{V}B\hat{Y}$ Matrix into Production Activities

$$Va' = \hat{V}BY = \underbrace{\hat{V}LY^D}_{(1)V\_D} + \underbrace{\hat{V}LY^F}_{(2)V\_RT} + \underbrace{\hat{V}LA^FLY^D}_{(3a)V\_GVC\_S} + \underbrace{\hat{V}LA^F(BY - LY^D)}_{(3b)V\_GVC\_C}$$

$$Y' = VB\hat{Y} = \underbrace{VL\hat{Y}^D}_{(1)Y\_D} + \underbrace{VL\hat{Y}^F}_{(2)Y\_RT} + \underbrace{VLA^FL\hat{Y}^D}_{(3a)Y\_GVC\_S} + \underbrace{VLA^F(B\hat{Y} - L\hat{Y}^D)}_{(3b)Y\_GVC\_C}$$

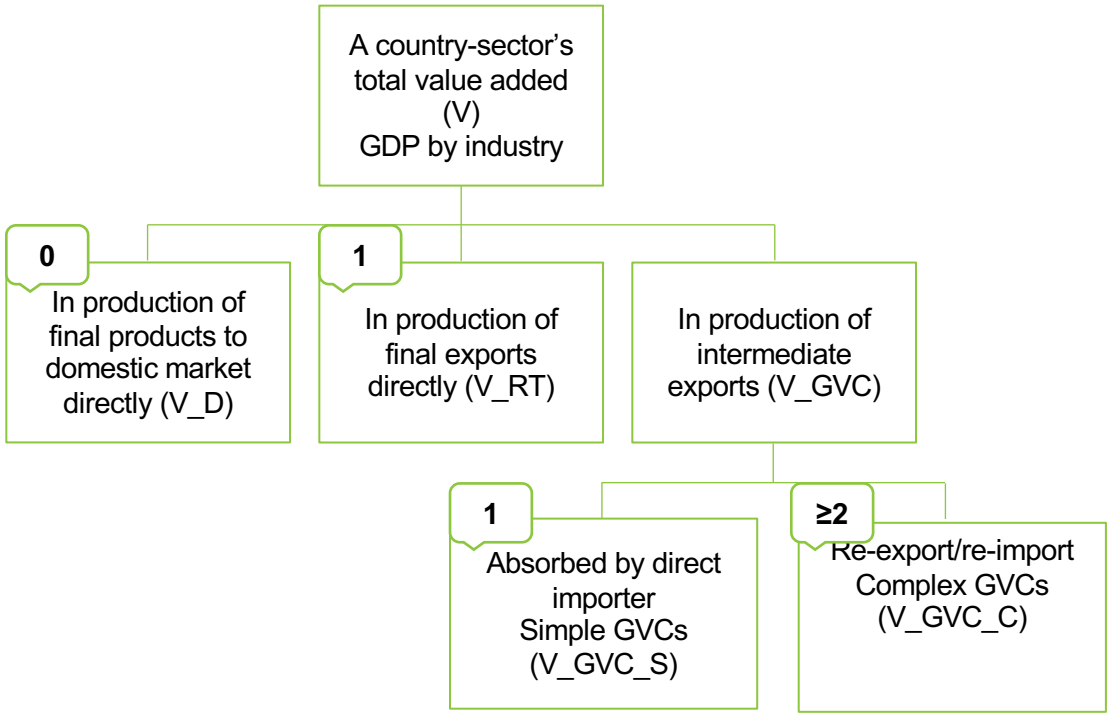
- V_D and Y_D represent value added produced at home and absorbed by domestic final demand without involving international trade.
- V_RT and Y_RT are domestic value added embodied in final product exports.
- V_D and V_RT are the sum of value added from a country-sector used in all downstream sectors.
- Y_D and Y_RT are the value added in a country sector that sums up the value added from all upstream sectors.
- In general, V_D and V_RT and different from Y_D and Y_RT except at the country aggregate level.
- V_RT + V_GVC_S + V_GVC_C = DVA_F (as in Koopman, Wang, and Wei 2014)
- Y_GVC_S + Y_GVC_C – RDV = FVA in the exporting country's final goods production (as defined by Los, Timmer, and de Vries 2015)

ADB

Decomposition of Value Added and Final Goods Production

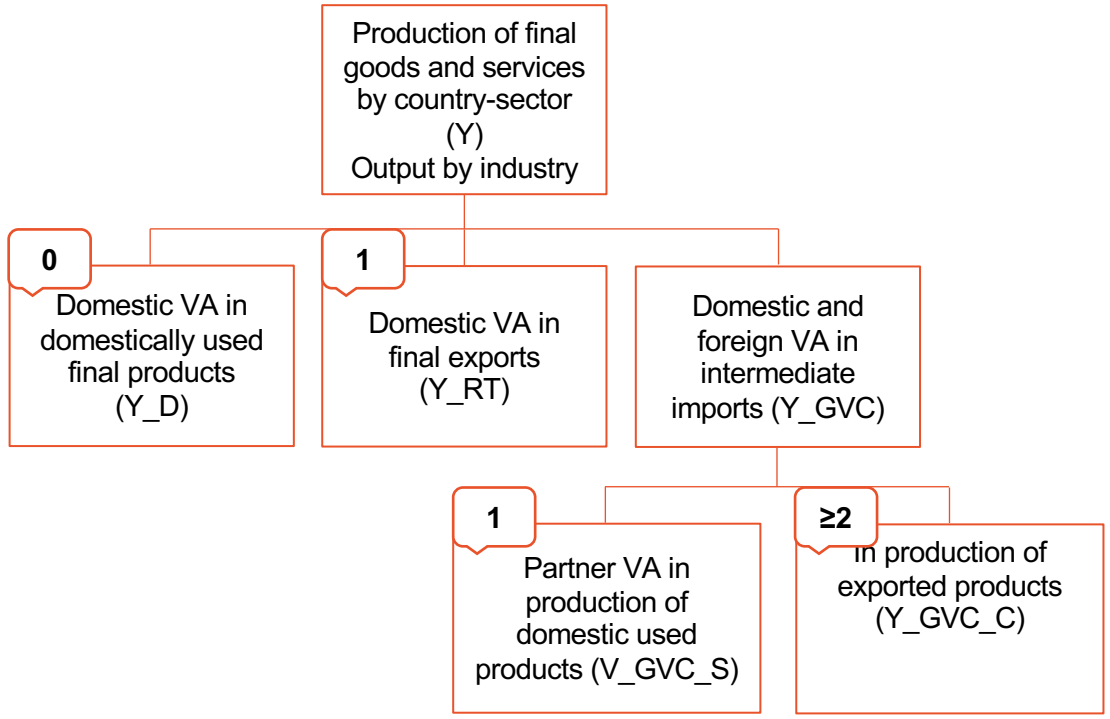
Decomposition of GDP by Country-Sector

(Which types of production and trade are GVC activities?)



Decomposition of Final Goods Production by Country-Sector

(Which part of final goods production and trade belong to GVCs?)



Global Value Chain Participation Index

- Compared to VS1 and VS as proposed by Hummels et al. (2001):
  - GVCPt_f and GVCPt_b are production concept instead of trade.
  - GVCPt_f is based on value added rather than gross exports.
  - GVCPt_b is a net concept rather than a gross concept.

- The relative values of the two indices indicate a country-sector's position in the global production network.
  - If GVCPt_f > GVCPt_b → the country-sector is more actively engaged in upstream production activities in GVCs.

Upstreamness

Antràs, P., Chor, D., Fally, T., and Hillberry, R. (2012). Measuring the upstreamness of production and trade flows. American Economic Review: Papers & Proceedings, 102(3), 412-416.

Antràs, P. and Chor, D. (2017).

Upstreamness

- Upstreamness – a measure of distance from final demand.
- Upstreamness aggregates information on the extent to which an industry in a given country produces goods that are sold directly to final consumer or that are sold to other sector that themselves sell disproportionately to final consumers.
- A relatively upstream sector is thus one that sells a small share of its output to final consumers, and instead sells disproportionately to other sectors that themselves sell relatively little to final consumers.

Upstreamness

- The gross output of sector r from country i (X_i^r) is equal to the sum of its use as a final good (Y_{ij}^r) and its use as intermediate input to other industries (Z_{ij}^{rs}):

$$X_i^r = \sum_{s=1}^{S}\sum_{j=1}^{J} Z_{ij}^{rs} + \sum_{j=1}^{J} Y_{ij}^r = \sum_{s=1}^{S}\sum_{j=1}^{J} Z_{ij}^{rs} + Y_i^r$$

- Define $a_{ij}^{rs} = Z_{ij}^{rs}/X_j^s$ as the dollar among of sector r's output from country i needed to produce one dollar worth of industry s's output in country j.

$$X_i^r = \sum_{s=1}^{S}\sum_{j=1}^{J} a_{ij}^{rs} X_j^s + Y_i^r$$

- Iterating this identity, we can express sector r's output in country i as an infinite sequence of terms, which reflect the use of this country-sector's output at different positions in the value chain, starting with its use as a final goods/service in all countries and sectors, as a direct input of a direct input in the production of final goods/services in all countries and industries, and so on:

$$X_i^r = Y_i^r + \sum_{s=1}^{S}\sum_{j=1}^{J} a_{ij}^{rs} Y_j^s + \sum_{s=1}^{S}\sum_{j=1}^{J}\sum_{t=1}^{S}\sum_{k=1}^{J} a_{ij}^{rs} a_{jk}^{st} Y_k^t + \cdots$$

Upstreamness

$$X_i^r = Y_i^r + \sum_{s=1}^{S}\sum_{j=1}^{J} a_{ij}^{rs} Y_j^s + \sum_{s=1}^{S}\sum_{j=1}^{J}\sum_{t=1}^{S}\sum_{k=1}^{J} a_{ij}^{rs} a_{jk}^{st} Y_k^t + \cdots$$

- Building on this identity, Antràs and Chor (2011) suggest computing the weighted average position of a country-sector's output in the global value chain by multiplying each of the terms in the above equation by its respective production-staging distance from final use plus one and dividing by X_i^r:

$$U_i^r = 1 \cdot \frac{Y_i^r}{X_i^r} + 2 \cdot \frac{\sum_{s=1}^{S}\sum_{j=1}^{J} a_{ij}^{rs} Y_j^s}{X_i^r} + 3 \cdot \frac{\sum_{s=1}^{S}\sum_{j=1}^{J}\sum_{t=1}^{S}\sum_{k=1}^{J} a_{ij}^{rs} a_{jk}^{st} Y_k^t}{X_i^r} + \cdots$$

ADB

Upstreamness

$$U_i^r = 1 \cdot \frac{Y_i^r}{X_i^r} + 2 \cdot \frac{\sum_{s=1}^{S}\sum_{j=1}^{J} a_{ij}^{rs} Y_j^s}{X_i^r} + 3 \cdot \frac{\sum_{s=1}^{S}\sum_{j=1}^{J}\sum_{t=1}^{S}\sum_{k=1}^{J} a_{ij}^{rs} a_{jk}^{st} Y_k^t}{X_i^r} + \cdots$$

- It is clear that $U_i^r \geq 1$ and that larger values are associated with relatively higher levels of upstreamness of the output originating from sector r in country i.
- Expanding the above and computing for all U_i^r, we have:

$$U = \frac{(1 + I + 2\times A + 3\times A^2 + 4\times A^3 + \cdots)Y}{X}$$

- Recall: $[I + A + A^2 + A^3 + \cdots] = [I - A]^{-1}$

$$U = \frac{[I - A]^{-1}[I - A]^{-1}Y}{X}$$

- Recall: $[I-A]^{-1}= B$ and $X = [I-A]^{-1}Y$
- The upstreamness of each sector to the final market is:

$$U = \frac{[I-A]^{-1}[I-A]^{-1}Y}{[I-A]^{-1}Y} = \frac{BBY}{BY}$$

- Provided that $\sum_{s=1}^{S}\sum_{j=1}^{J} a_{ij}^{rs} < 1$ for all j-s pairs, the numerator is equal to the $\big((i-1)\times S + r\big)$-th element of the $J\times S$ by 1 column matrix $[I-A]^{-2}Y$ or BBY.

- The upstreamness of each sector to a specific final product

$$U = \frac{[I-A]^{-1}[I-A]^{-1}\hat{Y}}{[I-A]^{-1}\hat{Y}} = \frac{BB\hat{Y}}{B\hat{Y}} = \frac{BB}{B}$$