Module 2: Review of Basics of Sampling Methods
Session 2.3: Systematic Sampling

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Topics Covered

* Different Types of Systematic Sampling
  * Linear systematic sampling
    * With $N/n = \text{Integer}$
    * Without $N/n = \text{integer}$
  * Circular systematic sampling
Systematic Sampling (SYS), like SRS, involves selecting $n$ sampling units from a population of $N$ units.

Instead of randomly choosing the $n$ units in the sample, a skip pattern is run through a list (frame) of the $N$ units to select the sample.

The skip or sampling interval, $k = N/n$. 
Linear systematic sampling: Selection process

1) Form a **sequential list** of population units

2) Decide on a sample size $n$ and compute the skip ($sampling\ interval$),$ k = N/n$

3) Choose a random number, $r\ (random\ start)$ between 1 and $k$ (inclusive)

4) Add “$k$” to selected random number to select the second unit and continue to add “$k$” repeatedly to previously selected unit number to select the remainder of the sample
Linear systematic sampling: Selection process

Sample Interval (k = N/n)

1 2 ..... r ..... k ..... n₂ ..... n₃ ..... etc ..... N

Sample Intervals

r = Random Start between 1 and k = n₁
Previous example assumed that $k = \frac{N}{n}$ is an integer

Question: What if $k = \frac{N}{n}$ is NOT an integer?

Solution 1: Work with decimal places and round

Solution 2: Circular sampling
### Example – working with decimals and rounding

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>N</td>
<td>32</td>
</tr>
<tr>
<td>n</td>
<td>7</td>
</tr>
<tr>
<td>skip</td>
<td>4.571429</td>
</tr>
<tr>
<td>R.Start</td>
<td>2.636695</td>
</tr>
</tbody>
</table>

- \( \frac{32}{7} \)
- \( \text{RAND() \times 4.571429} \)
- Same as the random start
- \( \text{roundup}(2.636695, 0) \)
1) Determine the interval $k$ – rounding down to the integer nearest to $N/n$
   (If $N = 15$ and $n = 4$, then $k$ is taken as $3$ and not $4$)

2) Take a random start between 1 and $N$

3) Skip through the circle by $k$ units each time to select the next unit until $n$ units are selected

4) Thus there could be $N$ possible distinct samples instead of $k$
Circular sampling illustration

Population = 24, Sample = 5, Skip = Int(24/5=4.6) = 4

Random start

1 2 3 4 5 6 7 8

24

23

22

21

Random start

17

20 19 18 17 16 15 14 13
Estimation with Systematic Sampling

The weight for a systematic sample is the same as Simple Random Sampling

Estimate of a total

\[ \hat{Y} = N \times \sum_{i=1}^{n} \frac{y_i}{n} = \sum_{i=1}^{n} \frac{N}{n} y_i = \sum_{i=1}^{n} w y_i \]

Estimate of a mean

\[ \hat{\bar{Y}} = \sum_{i=1}^{n} \frac{y_i}{n} \]
Estimation with Systematic Sampling (cont)

*Estimate of a proportion*

\[ y_i = \begin{cases} 1 & \text{ith sample unit has characteristic} \\ 0 & \text{otherwise} \end{cases} \]

\[ \hat{Y} = \frac{N}{n} \sum_{i=1}^{n} y_i = \hat{N}_c \]

\[ \bar{Y} = \frac{1}{n} \sum_{i=1}^{n} y_i = \hat{P}_c \]