Comparing CGE and NQT models: a formal overview of the model structures*

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Keywords: Computable General Equilibrium, New Quantitative Trade Models, GTAP
JEL codes: C68, F11, F13

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1 Introduction

In the current international trade literature quantitative trade models can be divided into computable general equilibrium (CGE) on the one hand and new quantitative trade (NQT) models on the other hand. CGE models are employed since more than 30 years to evaluate the effects of trade policies in a setting with multiple sectors featuring intermediate linkages. Dixon et al. (1982) is one of the first CGE models building on the pioneering work of Johansen (1960) and combining microfoundations of profit- and utility-maximization with an input-output structure describing the economy. More recently, so-called new quantitative trade models (Costinot and Rodriguez-Clare (2013)) have been introduced to analyse the welfare implications of trade and trade policies. New quantitative trade models emerge on the one hand from structural gravity (SG) models (Anderson and Van Wincoop (2003), Yotov et al. (2016)) and on the other hand from models employing exact hat algebra (EHA), introduced by Dekle et al. (2008). There are four main differences between CGE models on the one hand and NQT models on the other hand (see also Bekkers (2018)). First, CGE models are more extensive, containing more features to describe the economic and institutional details of international trade, whereas NQT models are more compact and parsimonious models. Second, structural gravity (SG) models differ from CGE models in the way the baseline is calibrated. SG models calibrate the baseline to predicted values of the estimated model, whereas CGE models calibrate the baseline to actual shares in the data. In terms of baseline calibration the two types of NQT models do not use the same approach, since models applying exact hat algebra follow the CGE tradition and calibrate the baseline to actual shares. Third, NQT models are (or claim to be) more rigorous on the requirement to apply structural estimation, i.e. deriving the estimating equations from exactly the same model as the model used for counterfactual experiments and estimating all parameters with the same dataset as the dataset used for the counterfactual experiments. CGE models are more flexible and take more often parameters from the literature. Fourth, the different literatures solve counterfactual experiments in different ways. In the CGE-literature there are two approaches. CGE-in-levels, using GAMS-software, solves the model in levels for both baseline values and counterfactual values, comparing the differences. CGE-in-relative-changes, using GEMPACK-software, solves the model in percentage changes, thus directly calculating changes as a result of a counterfactual experiment. SG-models, using STATA and Matlab follow the CGE-in-levels approach and calculate the equilibrium both for baseline and counterfactual values, whereas models applying exact hat algebra, using Matlab, follow the CGE-in-relative-changes approach and calculate directly ratios of new and old equilibrium values.

This paper compares the structure of two types of models in the quantitative trade literature: computable general equilibrium (CGE) models and new quantitative trade (NQT) models. The formal structure of the most well-known CGE-model, GTAP, is outlined into detail and compared with the structure of the principal references in the NQT literature: Caliendo and Parro (2015) and Costinot and Rodriguez-Clare (2013).

In this paper we focus on the first difference between the two approaches. In particular, this paper compares the structure of two types of models in the quantitative trade literature (CGE-models and NQT-models) by outlining the formal structure of the most well-known CGE-model, GTAP, into detail and comparing it with the structure of the principal references in the NQT literature: Caliendo and Parro (2015) and Costinot and Rodriguez-Clare (2013). The ramifications of differences in baseline calibration are discussed at length in Bekkers (2018). That paper also maps out into detail the differences in solution methods. The degree of rigour in structural estimation does not have an impact on the effects of trade policy experiments and is therefore not further discussed. The current paper maps out in turn the structure of demand, production, international trade, savings and investment, transport services, tax revenues, goods market equilibrium, and concludes with a discussion of the different macroeconomic closures.

Five differences between CGE models and NQT models are identified. In particular, CGE models contain the following elements, which are typically absent in NQT models: (i) Different import demand shares by end user (private households, government, and firms); (ii) Savings, investment, and capital; (iii) Non-homothetic preferences in private household demand; (iv) Substitution elasticities deviating from 1 in the choice between intermediates and between factors of production; (v) Export subsidies, other tax instruments, and a separate transport sector. The paper attempts to provide a complete overview of the structure of the CGE-model GTAP, formally mapping out all equilibium conditions. It does not go into the implementation in different software packages used for CGE-modelling such as GEMPACK and GAMS. Excellent overviews of GTAP focusing on the implementation in the different softwares are for example Hertel (1997) and Corong et al. (2017) for GEMPACK and Lanz and Rutherford (2016) and Britz and Van der Mensbrugghe (2015) for GAMS.

2 The Formal Structure of a Typical CGE-Model

2.1 Introduction

This section outlines the formal structure of the CGE-model GTAP into detail, describing in turn demand, production, savings and investment, transport services, goods market equilibrium and the definition of income, and tax revenues.[1] At the end of each subsection the differences with the equilibrium conditions in NQT models are discussed. It concludes with an overview of the endogenous variables and equilibrium equations. There is an economy with $j = 1,..,J$ countries and $r = 1,..,R$ sectors. A representative consumer buys with her income three categories of goods, private goods, government goods, and savings. Savings are channeled to a global bank allocating savings across the different countries, thus buying investment goods, q_j^{in}, in the different countries. Investment is allocated across countries such that rates of return tend to equalize across countries. In order to produce, firms demand both intermediate inputs and factors of production. The choice between intermediate inputs and value added bundles is modelled as Leontief and the choice between different production factors is constant elasticity of substitution (CES). There are six production factors. The supply of land and natural resources are fixed. In the baseline model, labor supply and capital employed in production are fixed and not affected by investment by the global bank. Land and natural resources are typically

[1] The description of some parts of the model (production, transport services, and household income) is similar to the description in Bekkers et al. (2018)

imperfectly mobile across different sectors. International trade is modelled with an Armington structure featuring love-of-variety between goods from different countries of origin. The import price is equal to the export price plus the export tax, the cif-fob margin, and the import tariff. The cif-fob margin is used to pay the global transportation sector, which hires transport services from different countries. The model also features production, endowment, (direct) income, and value added taxes. In equilibrium the income of the representative household spent on the three types of goods consists of the sum of gross factor income and indirect tax revenues. In the standard setting the trade balance adjusts endogenously, but the trade balance can also be fixed by endogenizing either savings or investment.

2.2 Demand

A representative consumer in each country j spends her income on quantities of three categories of goods, private goods, q_j^{pr}, government goods, q_j^{go}, and savings, q_j^{sa}, according to a Cobb-Douglas utility function:

$$u_j = \left(q_j^{pr}\right)^{\kappa_j^{pr}} \left(q_j^{go}\right)^{\kappa_j^{go}} \left(q_j^{sa}\right)^{\kappa_j^{sa}} \tag{1}$$

Savings are included in the static utility function to prevent that a shift away from savings –and thus implicitly from future consumption– towards current consumption would have large welfare effects. In that case the corresponding changes in the trade balance could have large welfare effects. Including savings in the static utility function allows the model to account in a consistent way for an endogenous trade balance and for investment without making the model dynamic. The formal underpinning comes from Hanoch (1975) who showed that the expressions for consumption in an inter-temporal setting can also be derived from a static utility maximisation problem with savings in the utility function.

2.2.1 Homothetic Sectoral Demand

If the demand for government goods and private goods across the different sectors were homothetic, we could work with the following Cobb-Douglas expressions for demand on category c, q_j^c, and the price index of utility, p_j^u, as a function of the price of category c goods, p_j^c, and household income, x_i:

$$q_j^c = \frac{\kappa_j^c}{p_j^c} x_i; c = pr, go, sa \tag{2}$$

$$p_j^u = \left(\frac{p_j^{pr}}{\kappa_j^{pr}}\right)^{\kappa_j^{pr}} \left(\frac{p_j^{go}}{\kappa_j^{go}}\right)^{\kappa_j^{go}} \left(\frac{p_j^{sa}}{\kappa_j^{sa}}\right)^{\kappa_j^{sa}} \tag{3}$$

We can for example assume that sectoral demands of private and government goods are also Cobb-Douglas. This would imply the following expressions for the quantity of private and government goods, q_{js}^c, and for the corresponding price index, p_j^c:

$$q_{js}^c = pop_j \frac{\beta_{js}^c}{p_{js}^c} p_j^c q_j^c; c = pr, go \tag{4}$$

$$p_j^c = \prod_{s=1}^{S} \left(\frac{p_{js}^c}{\beta_{js}^s}\right)^{\beta_{js}^c}; c = pr, go \tag{5}$$

To obtain sectoral demand, we have multiplied per capita demand by the size of the population, pop_j, to move from per capita units to country-level units. Including non-homothetic preferences for private demand considerably complicates the exposition, as the upper-nest demands by the different groups of agents also have to be adjusted, as will be shown in the next subsection.

2.2.2 Non-Homothetic Sectoral Demand

In this subsection it is assumed that preferences for private goods are non-homothetic. This implies that it is not possible to define a price for private goods. Therefore, we cannot maximise utility in equation (1) subject to a conventional budget constraint. Instead we maximise utility in equation (1) subject to the following implicit budget constraint, with expenditures on category c goods, e_j^c, a function of the quantity of private consumption, q_j^c:

$$\sum_c e_j^c \left(q_j^c\right) = x_j. \tag{6}$$

This leads to the following expression for expenditures, x_j^c, on the three categories of goods, $c \in \{pr, go, sa\}$, as a function of total expenditure (x_j):

$$x_j^c = \kappa^c \left(\frac{\Psi_j^c}{\Psi_j}\right) x_j; c = pr, go, sa \tag{7}$$

where Ψ_j^c is the elasticity of quantity, q_j^c, with respect to expenditure, x_j^c, and Ψ_j is the elasticity of utility, u_j, with respect to total expenditure, x_j. For goods with homothetic preferences – savings (sa) and public goods (go)– this elasticity is 1 ($\Psi_j^{go} = \Psi_j^{sa} = 1$). So with homothetic preferences, equation (7) would generate the standard expression for Cobb-Douglas expenditure shares. With non-homothetic preferences for private goods the share of spending on private goods is larger than the Cobb-Douglas parameter κ^{pr} if the elasticity of private quantity, q_j^{pr}, with respect to private expenditure, x_j^{pr}, is larger than 1. This gives the consumer an incentive to spend a more than proportional amount on private goods.

Ψ_j^{pr} follows from log differentiating the indirect utility function for private goods defined below in equation (10) below with respect to quantity (q_j^{pr}) and expenditure (x_j^{pr}). This gives the following expression:

$$\Psi_j^{pr} = \frac{1}{\sum\limits_{s=1}^{S} s_{js}^{pr} \eta_{js}} \tag{8}$$

where s_{js}^{pr} is the share of private expenditure spent on good s. Ψ_j follows from maximisation of utility in equation (1):[2]

$$\Psi_j = \sum_c \Psi^c \kappa^c = \Psi^{pr} \kappa^{pr} + \kappa^{go} + \kappa^{sa}. \tag{9}$$

Preferences for private goods across the different sectors are described by the non-homothetic Constant Distance Elasticity (CDE) implicit expenditure function:

$$\sum_{s=1}^{S} \alpha_{js} \left(q_j^{pr} \right)^{\gamma_{js}\eta_{js}} \left(\frac{p_{js}^{pr}}{x_j^{pr}} \right)^{\gamma_{js}} = 1 \tag{10}$$

where q_{js}^{pr} and p_{js}^{pr} are respectively the quantity and price of private goods in country j and sector s, x_j^{pr} is private expenditure in country j, while α_{js}, γ_{js} and η_{js} are respectively the distribution, substitution and expansion parameters. Private demand, q_{js}^{pr}, as a function of private expenditure, x_j^{pr}, and prices, p_{js}^{pr}, can be derived by log-differentiating equation (10) with respect to p_{js}^{pr} and x_j^{pr} and applying Shepherd's lemma:

$$q_{js}^{pr} = \frac{\alpha_{js} \left(q_j^{pr} \right)^{\gamma_{js}\eta_{js}} \left(\frac{p_{js}^{pr}}{x_j^{pr}} \right)^{\gamma_{js}-1} \gamma_{js}}{\sum\limits_{u=1}^{S} \alpha_{ju} \left(q_j^{pr} \right)^{\gamma_{ju}\eta_{ju}} \left(\frac{p_{ju}^{pr}}{x_j^{pr}} \right)^{\gamma_{ju}} \gamma_{ju}}. \tag{11}$$

With CDE preferences the model allows for shifting average and marginal budget shares as

[2]See Mcdougall (2000) for further discussion and Appendix Appendix B for a formal derivation.

a country grows. At the same time the model stays tractable in a setting with a large number of countries and sectors, since a limited number of parameters can be calibrated from income and own-price elasticities of demand. As such CDE preferences are a compromise between a more basic way to model non-homothetic preferences, like the linear expenditure system and a more extensive way like AIDADS, requiring a large number of parameters.

Preferences for spending by the public sector across the different sectors continue to be Cobb-Douglas, so equations (4)-(5) hold for demand for government goods. For public goods and savings, quantity and expenditure are simply related by the following expression, $q_j^c = \frac{x_j^c}{p_j^c}; c = go, sa$. For private goods we cannot define a price index. Quantity, q_j^{pr}, and expenditure, x_j^{pr}, are implicitly related through the indirect expenditure function in (10).

2.2.3 Differences with NQT Models

The NQT models do not distinguish between different end users, do not include savings, and work with homothetic preferences across sectors, typically Cobb-Douglas preferences as in Subsection 2.2.1. Not distinguishing between end users means that spending shares across sectors are identical for government goods and private goods. Given that both types of spending enter the utility function in the GTAP-model this will not make a big difference. Not including savings means that it is hard to model investment and changes in the trade balance. Non-homothetic preferences are especially useful for dynamic simulations with rising incomes provoking changes in spending shares.

2.3 Production

Input bundles used in production, q_{is}^{ib}, are a Leontief function of factor input (value added) bundles, q_{is}^{va}, and intermediate input bundles bought by firms in sector s (with superscript fi_s) from sector r, $q_{ir}^{fi_s}$:

$$q_{is}^{ib} = \min\left\{\varpi_{is}^{va} q_{is}^{va}, \varpi_{i1}^{fi_s} q_{i1}^{fi_s}, .., \varpi_{iS}^{fi_s} q_{iS}^{fi_s}\right\} \tag{12}$$

The demand for value added and intermediates are defined as:

$$q_{is}^{va} = \varpi_{is}^{va} q_{is}^{ib} \tag{13}$$

$$q_{ir}^{fi_s} = \varpi_{ir}^{fi_s} q_{is}^{ib} \tag{14}$$

Intermediate input demand by firms from sector s for goods from sector r, $q_{ir}^{fi_s}$, coresponds with the demand by agent $ag = fi_s$, q_{ir}^{ag}, in equation (23).[3]

The price of input bundles, p_{is}^{ib}, is a function of the prices of intermediates, p_{ir}^{fi}, and the price of value added p_{is}^{va}:

$$p_{is}^{ib} = \varpi_{is}^{va} p_{is}^{va} + \sum_{r=1}^{S} \varpi_{irs}^{int} p_{ir}^{fi_s} \tag{15}$$

Demand for the different production factors is CES, implying the following expressions for the price of value added, p_{is}^{va}, and demand for factor inputs (or endowments e), q_{ise}^{end}:

$$p_{is}^{va} = \left[\sum_{e=1}^{E} (\iota_{ise})^{\chi_s} \left(t_{ise}^{end} \omega_{ise} \right)^{1-\chi_s} \right]^{\frac{1}{1-\chi_s}} \tag{16}$$

$$q_{ise}^{end} = \left(\iota_{ise} \frac{c_{is}^{va}}{t_{ise}^{end} \omega_{ise}} \right)^{\chi_s} q_{is}^{va} \tag{17}$$

ω_{ise} and t_{ise}^{end} are respectively the price of and the tax on endowment e, and χ_s is the substitution elasticity between production factors. With this specification there is one substitution elasticity between the different factors of production.[4]

Since capital ca is mobile across sectors, its factor price w_{ica} (also the nominal rental rate on capital) is equal between sectors:

$$\omega_{isca} = w_{ica} \tag{18}$$

Equality of capital supply, q_{ica}^{end}, and capital demand corresponds with:

$$q_{ica}^{end} = \sum_{s=1}^{S} q_{isca}^{end} \tag{19}$$

Land and natural resources are imperfectly mobile across sectors, modelled by the following elasticity of transformation function:

$$q_{ie}^{end} = \left[\sum_{s=1}^{S} \vartheta_{ise} \left(q_{ise}^{end} \right)^{\frac{\mu_e+1}{\mu_e}} \right]^{\frac{\mu_e}{\mu_e+1}} \quad ; e = ld, nr, ls, ms, hs \tag{20}$$

[3]To be able to define demand for all groups of agents ag in the description of international trade we have chosen to work with the somewhat akward notation for intermediate demand, $q_{ir}^{fi_s}$, with the subscript r indicating the sector from which intermediates are bought and the subscript s in the superscript fi_s indicating the sector buying the intermediates.

[4]Many other CGE-models feature a more nested structure where firms choose for example first between a composite of high-skilled labor and capital on the one hand and low-skilled labor on the other hand in order to model skill-capital complementarity. Or the intermediate energy could be modelled as a composite together with capital.

q_{ie}^{end} is the total quantity of immobile factor d and q_{ise}^{end} the quantity used in sector s. Also labor could be modelled as imperfectly mobile across sectors, a useful feature in models with imperfect competition with the risk of corner solutions.The supply of the immobile production factors to the different sectors, q_{ise}^{end}, can thus be expressed as follows:

$$q_{ise}^{end} = \left(\frac{\omega_{ise}}{\vartheta_{ise} w_{ie}}\right)^{\mu_e} q_{ie}^{end}; e = ld, nr, ls, ms, hs \tag{21}$$

So factor price differences across sectors are possible and q_{ise}^{end} moves to the sector where the price w_{ise} is higher. w_{ie} is the average price of the immobile factor w_{ie} across sectors:

$$w_{ie} = \left(\sum_{i=1}^{N} (\vartheta_{ise})^{-\mu_d} (\omega_{ise})^{\mu_e+1}\right)^{\frac{1}{\mu_e+1}}; e = ld, nr, ls, ms, hs \tag{22}$$

The supply of capital is described in the next subsection. The supply of the other factors of production is fixed, although it is possible to include for instance endogenous labor supply as for example in Bekkers et al. (2018).

Caliendo and Parro (2015) include only labor as production factor in their model, whereas Costinot and Rodriguez-Clare (2013) consider the possibility of multiple factors of production. Imperfect labor mobility is included in recent work by Caliendo et al. (2015). NQT models typically assume Cobb-Douglas nests between value added and intermediates and between different intermediates, whereas CGE models work with Leontief nests thus limiting the scope for substitution between production factors.

2.4 International Trade

International trade is modelled with Armington preferences in a nested structure. Hence, import demand features CES preferences between domestic and imported goods and in turn between imports from different sourcing countries. So, goods from different countries are differentiated. Typically, the substitution elasticity between imports and domestic goods is smaller than between imports from different sources. The group of agents ag in country j divides demand within each sector, q_{js}^{ag}, between demand for domestic and imported goods, $q_{js}^{d,ag}$ and $q_{js}^{m,ag}$, according to a CES utility function:

$$q_{js}^{ag} = \left(\left(q_{js}^{d,ag}\right)^{\frac{\rho_s-1}{\rho_s}} + \left(q_{js}^{m,ag}\right)^{\frac{\rho_s-1}{\rho_s}}\right)^{\frac{\rho_s}{\rho_s-1}} \tag{23}$$

Quantities of imported and domestic varieties can be summed across the groups of agents to give total importer and domestic demand, q_{js}^{so} with superscript so the source, $so = d, m$:

$$q_{js}^{so} = \sum_{ag} q_{js}^{so,ag} = \sum_{ag} \left(\frac{ta_{js}^{so,ag} p_{js}^{so}}{p_{js}^{ag}} \right)^{-\rho_s} q_{js}^{ag} \tag{24}$$

$ta_{js}^{so,ag}$ is a group–specific and source-specific tariff, expressed in power terms, so as 1 plus the ad-valorem tariff rate. p_{js}^{ag} and p_{js}^{so} are respectively the prices corresponding with q_{js}^{ag} and q_{js}^{so}. Since q_{js}^{so} is homogeneous across the different agents it does not have a superscript ag. In the data this setup corresponds with the availability of group-specific aggregate import and domestic spending shares, but the absence of group-specific information on trade-partner-specific import shares. So the data provide for example information on specific import and domestic spending shares of the government, private households, and each of the sectors demanding intermediates, but not on import shares from different trading partners. National accounts provide group-specific import shares, but in international trade data group-specific import shares per trading partner are absent.

Import demand, q_{js}^{m}, is in turn distributed across imports from different sourcing countries i, q_{ijs}:[5]

$$q_{js}^{m} = \left(\sum_{i=1}^{J} (q_{ijs})^{\frac{\sigma_s - 1}{\sigma_s}} \right)^{\frac{\sigma_s}{\sigma_s - 1}} \tag{25}$$

Import demand from specific source countries i, q_{ijs}, is thus equal to:

$$q_{ijs} = \left(\frac{p_{ijs}}{p_{js}^{m}} \right)^{-\sigma_s} q_{js}^{m} \tag{26}$$

p_{js}^{ag}, p_{js}^{m}, and p_{ijs} are respectively the prices corresponding with q_{js}^{ag}, q_{js}^{m}, and q_{ijs}, defined as

[5] To allow for intra-regional international trade the summation includes country j. Intra-regional international trade is relevant when regions in the model are aggregates of different countries.

follows:

$$p_{js}^{ag} = \left(\left(ta_{js}^{d,ag} p_{js}^{d} \right)^{1-\rho_s} + \left(ta_{js}^{m,ag} p_{js}^{m} \right)^{1-\rho_s} \right)^{\frac{1}{1-\rho_s}} \tag{27}$$

$$p_{js}^{d} = tp_{js} b_{js} p_{js}^{ib} \tag{28}$$

$$p_{js}^{m} = \left(\sum_{i=1}^{J} p_{ijs}^{1-\sigma_s} \right)^{\frac{1}{1-\sigma_s}} \tag{29}$$

$$p_{ijs} = ta_{ijs} t_{ijs} p_{ijs}^{cif} = ta_{ijs} t_{ijs} \left(te_{ijs} tp_{is} b_{is} p_{is}^{ib} + \frac{p_{ijs}^{ts}}{a_{ijs}^{ts}} \right) \tag{30}$$

The price of the domestic good is equal to the marginal cost, b_{js}, times the production tax, tp_{is}, times the input bundle price, p_{is}^{ib}, divided by the domestic supply shifter, c_{js}^{d}. The price of the traded good, p_{ijs}, in equation (30) is equal to the cif-price, p_{ijs}^{cif} times iceberg trade costs, t_{ijs}, times bilateral ad valorem tariffs, ta_{ijs}, both expressed in power terms. The cif-price, p_{ijs}^{cif}, in turn is calculated as marginal cost, b_{is}, times the production tax, tp_{is}, times the price of input bundles in the exporting country, p_{is}^{ib}, times the export subsidy applied to the fob-price, te_{ijs}, plus the price of transport services, p_{ijs}^{ts}, divided by a transport services technology shifter, a_{ijs}^{ts}. Firms spend a fixed quantity share of sales on transport services.

Many NQT work also with Armington preferences, especially applications in the structural gravity literature. In NQT models using exact hat algebra, the Eaton and Kortum (2002)-structure is employed a lot. As shown in Arkolakis, et al. (2012) the welfare gains from trade are identical under Armington and Eaton-Kortum.[6] Costinot and Rodriguez-Clare (2013) include monopolistic competition both with identical and heterogeneous firms in NQT models, whereas Caliendo et al. (2017) study the impact of tariffs under firm heterogeneity in an NQT model. The Armington trade structure has been extended in the CGE literature with monopolistic competition with identical firms as in Ethier (1982) and Krugman (1980) and heterogeneous firms as in Melitz (2003) (see respectively Francois (1998), Swaminathan and Hertel (1996) and Zhai (2008), Balistreri (2012), Akgul et al. (2016), Dixon (2016), and Bekkers and Franois (2017)). Both the CGE and NQT literature concludes that the welfare effects of trade cost reductions are larger under firm heterogeneity, but there is no consensus on how much larger the effects are.

NQT models impose identical domestic and import shares for final foods and intermediates, whereas CGE-models allow for differences in import shares based on information in national

[6] Bekkers et al. (2018) have included an Eaton-Kortum trade structure in a CGE model.

accounts data. One step further would be to allow for different source-specific import shares for different end users. Koopman et al. (2013) have calculated such import shares based on the distinction between intermediate and final goods in the UN classification of Broad economic categories (BEC) in order to improve trade in value added statistics. CGE models with different source-specific import shares for different end users are currently developed (see Walmsley et al. (2014)).

2.5 Savings and Investment

In the standard GTAP model the amount of capital used in production is exogenous. There is investment in the model, but additional capital as a result of investment does not contribute to the employed capital stock, i.e. does not come online. To model this we distinguish between the so-called beginning-of-period capital stock, used in production, and the end-of-period capital stock, affected by investment flows. As discussed at the end of this section this approach can be easily extended such that additional capital as a result of investment does come online and is thus used in production.

To model international capital flows the concept of a global bank is used. All savings are channeled to the global bank, which invests the savings funds across different regions with rates of return tending to equalize. Because of capital adjustment costs, rates of return tend to equalize but do not actually equalize. The price of savings in a specific country is a weighted average of the price of investment goods across the world acquired by the global bank and the price of domestic investment goods. With this specification more weight is given to the domestic price of investment flows. Both capital adjustment costs and the additional weight on the price of domestic investment goods in the price of savings attempt to capture the empirical regularity that savings and investment move together at the national level.

To model this setup formally, we can start with the relation between the beginning-of-period capital stock, kb_i, investment, q_i^{in}, and the end-of-period capital stock, ke_i, according to the following equation with δ_i the rate of capital depreciation:

$$ke_i = (1 - \delta)\, kb_i + q_i^{in} \tag{31}$$

The beginning-of-period capital stock is used in production, thus giving:

$$q_{ica}^{end} = kb_i \tag{32}$$

Investment goods are like intermediates a Leontief composite of goods used for investment from different industries, q_{is}^{in}:

$$q_i^{in} = \min\left\{\varpi_{i1s}^{in} q_{i1}^{in}, .., \varpi_{iS}^{in} q_{iS}^{in}\right\} \tag{33}$$

Investment demand from different sectors, q_{is}^{in}, and the aggregate price of investment goods, p_i^{in}, are thus defined as:

$$q_{is}^{in} = \varpi_{is}^{in} q_i^{in} \tag{34}$$

$$p_i^{in} = \sum_{s=1}^{S} \varpi_{is}^{in} p_{is}^{in} \tag{35}$$

Total investment demand, q_i^{in}, is determined by the condition that rates of return tend to equalize between countries, modelled by distinguishing between the current and expected rate of return. The current real rate of return on capital, r_i, can be different across countries. It is defined as the rental rate on capital, w_{ica}, divided by the price of investment goods, p_i^{in}, minus the rate of depreciation, δ_i:

$$r_i = \frac{w_{ica}}{p_i^{in}} - \delta_i \tag{36}$$

The expected real rate of return, r^e, instead is equalized across different countries. r^e is proportional to the current rate of return, but is scaled down by a factor proportional to the ratio of the end-of-period to beginning-of-period capital stock, reflecting the presence of capital adjustment costs:[7]

$$r^e = r_i \left(\frac{ke_i}{kb_i}\right)^{-flex} \tag{37}$$

$flex$ is a parameter determining the importance of capital adjustment costs. A large value of $flex$ means that additions to the capital stock, corresponding with $ke_i > kb_i$, are costly and thus have a strong negative impact on the expected rate of return. As a result only relatively small flows of capital already lead to equalization of the expected rate of return. With small values of $flex$ instead large changes in the capital stock are required to equalize rates of return.

Total investment, q_{in}, is thus determined by equations (31), (36), and (37), together with

[7] r^e is determined because one of the prices in the model is set as numeraire.

the demand for capital in production, as specified in equation (19). A shock raising demand for capital would increase the nominal rental rate w_{ica}. This in turn leads to an increase in the end-of-period capital stock, so both r_i and ke_i rise according to equations (36)-(37). The rise in ke_i in turn generates an increase in investment demand q_i^{in}. The degree to which an increasing rate of return on capital, r_i, leads to additional investment and thus capital inflows into a country is determined by $flex$. A small value of $flex$ generates a larger response in ke_i and thus a larger investment response.

The price of savings in country i is determined by the domestic price of investment and a weighted average of the global price of investment goods:

$$p_i^s = p_i^{in} \sum_{k=1}^{J} \left(p_k^{in}\right)^{\chi_k} \tag{38}$$

χ_k is the share of net investment in country k:

$$\chi_k = \frac{p_k^v \left(q_k^v - \delta_k q_{kca}^{end}\right)}{p^v \sum_{j=1}^{K} \left(q_j^v - \delta_j q_{jca}^{end}\right)} \tag{39}$$

In equilibrium the global value of savings is equal to the global value of net investments:[8]

$$\sum_{i=1}^{J} pop_i p_i^s q_i^s = \sum_{i=1}^{J} p_i^{in} \left(q_i^v - \delta q_{ica}^{end}\right) \tag{40}$$

To model endogenous capital accumulation, we assume that the beginning- and end-of-period capital stock are identical, $ke_i = kb_i$. This implies that current rates of return, r_i, are equalized across countries according to equation (37), as there can be no capital adjustment costs anymore. Hence, only equation (36) together with the following relation between capital, kb_i, and investment, q_i^{in}, will determine investment:

$$q_i^{in} = \frac{kb_i}{\delta_i} \tag{41}$$

A shock raising demand for capital would as before raise the nominal rental rate, w_{ica}. But it would also directly raise investment demand according to equation (41). This in turn would raise the price of investment goods, p_i^{in}, thus muting the initial impact of the higher nominal

[8] In the code (40) is the equation that can be omitted by Walras' law.

rate of return on the real rate of return, r_i, in equation (36).

Investment and capital accumulation have not been included in NQT models with the exception of the work by Anderson (2015). These scholars incorporate basic capital accumulation dynamics in a single sector setting without intermdiate linkages.

2.6 Transport Services

The cif-quantity q_{ijs} is a Leontief aggregate of the fob-quantity q_{ijs}^{fob} and the quantity of transport services ts_{ijs}:[9]

$$q_{ijs} = \min\left\{q_{ijs}^{fob}, \frac{ts_{ijs}}{a_{ijs}}\right\} \tag{42}$$

So there is no scope for substitution between transport services and fob-quantities and the quantity of transport services is proportional to the quantity traded:

$$ts_{ijs} = \gamma_{ijs} q_{ijs} \tag{43}$$

Equation (42) implies that the cif-price is additive in the fob-price and thus implies the expression for the price of transport services in equation (30).

Since transport services are a homogeneous good globally, there is only one price of transport services, p^{ts}, and the price of transport services used between i and j in sector s, p_{ijs}, is proportional with this price:

$$p_{ijs}^{ts} = p^{ts} \tag{44}$$

The demand for transport services on all routes is equal to the supply of global transport services, ts, provided by a global transport sector:

$$\sum_{s=1}^{S}\sum_{i=1}^{J}\sum_{j\neq i} ts_{ijs} = ts \tag{45}$$

The global transport sector demands transport services in turn from different countries supplying these services, according to a Cobb-Douglas function:[10]

$$p^{ts} = \prod_{i=1}^{J}\left(p_{its}^{ib}\right)^{\nu_i} \tag{46}$$

[9] We assume that there is only one type of transport sector, but this could easily be generalized.

[10] Because more detailed data on transport services are lacking, the supply of transport services first goes into a global transport sector, which in turn distributes these services without a link between the supplying country and the demanding trade partners

Table 1: *Overview taxes in the model*

Tax rate	**Tax revenue**	**Description**
tn_{ise}	tnr_{ise}	Tax on use of endowment e in sector s
td_{ie}	tdr_{ie}	Direct income tax on endowment e
tp_{is}	tpr_{is}	Tax on production in sector s
$ta_{js}^{so,ag}$	$tar_{js}^{so,ag}$	Tax on purchases by group $ag = p, g, f$ in sector s in sector s from source $so = imp, dom$
ta_{ijs}	tar_{ijs}	Tax on imports (tariff) from i to j in sector s
te_{ijs}^{exp}	ter_{ijs}	Tax on exports from i to j in sector s

By a slight abuse of notation the transport services sector has sector index ts. This sector also produces services sold domestically and to its trading partners. The corresponding demand for transport services from country i, ts_i, is given by:

$$ts_i = \varkappa_i \frac{p^{ts} ts}{p_{its}^{ib}} \tag{47}$$

A cif-fob margin and a transport sector are not part of the NQT literature.

2.7 Tax Revenues

Table 1 gives an overview of the six type of taxes in the model (taken from Bekkers et al. (2018)). Tax rates are in power terms (as one plus the ad valorem rate).

It is straightforward to calculate the tax revenues for the first three domestic tax rates, since tax bases are unambiguously defined:

$$tnr_{ise} = (tn_{ise} - 1)\, \omega_{ise} q_{ise}^{end} \tag{48}$$

$$tdr_{ie} = (td_{ie} - 1)\, w_{ie} q_{ie}^{end} \tag{49}$$

$$tpr_{is} = (tp_{is} - 1)\, p_{is}^{ib} q_{is}^{ib} \tag{50}$$

Source-specific import tax revenues, $t_{js}^{so,ag}$, can also be easily determined according to the following equation with $q_{js}^{so,ag}$ defined in (24):

$$tar_{js}^{so,ag} = \left(ta_{js}^{so,ag} - 1 \right) p_{js}^{so,ag} q_{js}^{so,ag} \tag{51}$$

Import tariff revenues and export tax revenues can also be expressed in a straightforward way as follows:

$$tar_{ijs} = \frac{(ta_{ijs} - 1)\, p_{ijs} q_{ijs}}{ta_{ijs}} \tag{52}$$

$$ter_{ijs} = (te_{ijs} - 1)\, b_{is} p_{is}^{ib} q_{ijs} \tag{53}$$

We observe that because of the way the bilateral price p_{ijs} is defined (tariff inclusive), we have to divide by the power tariff rate ta_{ijs} in equation (52) to calculate tariff revenues. This is not the case for source-specific import and domestic tax revenues and export taxes instead, as for these taxes the tax base is defined based on prices, exclusive of the power of the tax.

The different indirect tax revenues can be added up to generate total tax revenues on indirect taxes:

$$tir_i = \sum_{e=1}^{E} \sum_{s=1}^{S} tnr_{ise} + \sum_{s=1}^{S} \left[tpr_{is} + \sum_{ag} \sum_{so} tar_{is}^{so,ag} + \sum_{j=1}^{J} (tar_{jis} + ter_{ijs}) \right] \tag{54}$$

The NQT literature does not take other taxes into account besides tariffs.

2.8 Goods Market Equilibrium and Definition of Household Income

We can define goods market equilibrium for each sector s in country i by equalizing the quantity produced, q_{is}^{prod}, with the quantity demanded, consisting of domestic demand, import demand, and demand for transport services (in the transport services sector):

$$q_{is}^{ib} = \sum_{ag \in \{pr,go,fi,in\}} c_{is} q_{is}^{d,ag} + \sum_{j=1}^{J} c_{is} t_{ijs} q_{ijs} + ts_i;\ \ s = ts \tag{55}$$

$$q_{is}^{ib} = \sum_{ag \in \{pr,go,fi,in\}} c_{is} q_{is}^{ag} + \sum_{j=1}^{J} c_{is} t_{ijs} q_{ijs} + ts_i;\ \ s \neq ts \tag{56}$$

Per capita household expenditure is derived as follows. We use the macroeconomic accounting identity $Y = C + S + T$, following the exposition in Bekkers et al. (2018). So, gross factor income y_i (income before direct income taxes are paid) is equal to aggregate expenditures on savings, $pop_i x_i^{sa}$, plus expenditures on private consumption goods, $pop_i x_i^{pr}$, and payments of direct income taxes, tr_i^{dir}:[11]

$$y_i = pop_i\,(x_i^{sa} + x_i^{pr}) + tr_i^{dir} \tag{57}$$

Adding government spending on both sides and defining the budget deficit as $bd_i = pop_i x_i^{go} -$

[11] Payments of indirect taxes are part of consumption goods spending in this identity.

$\left(tr_i^{dir} + tr_i^{ind}\right)$ leads to:

$$y_i + tr_i^{ind} + bd_i = pop_i \left(x_i^{sa} + x_i^{pr} + x_i^{go}\right) \tag{58}$$

Combining these two equations implies that aggregate household expenditure $pop_i x_i$ on savings, private and government goods can be written as a function of income, y_i, indirect tax revenues, tr_i^{ind}, and the budget deficit ratio, bdr_i:

$$pop_i x_i = pop_i \left(x_i^{sa} + x_i^{pr} + x_i^{go}\right) = \left(y_i + tr_i^{ind}\right)\left(1 + bdr_i\right) \tag{59}$$

Income y_i consists of gross factor income of the different factors of production minus the value of depreciation of capital:

$$y_i = \sum_{e=1}^{E} t_{ie}^{inc} w_{ie} q_{ie}^{end} - \delta_i w_{ica}^{end} q_{ica}^{end} \tag{60}$$

bdr_i is defined as the budget deficit divided by gross factor income plus indirect tax revenues and assumed to be constant:

$$bdr_i = \frac{bd_i}{y_i + tr_i^{ind}} \tag{61}$$

2.9 Overview of Equilibrium Equations

In this section we give an overview of the endogenous variables and equilibrium equations. There are: J countries with subscripts i, j, and k; S sectors with subscripts r and s; three categories c of goods demanded by the representative agent: private goods, government goods, and savings with respectively superscripts pr, go, and sa; four groups of agents ag (or end users) buying goods: private consumers, the government, investors, and firms from each sector s demanding intermediates with respectively the superscripts pr, go, in, and fi_s; E factors of production with subscripts e; two sources so of goods, domestic d and imported m. Table 2 provides an overview of all equilibrium variables. Table 3 lists all equilibrium equations together with the complementary variables corresponding to each equation.

2.10 Macroeconomic Closures

The standard model imposes no restrictions on the trade balance and assumes fixed employment. The so-called closure of the model determines the set of endogenous and exogenous variables of the model and so in the closure these assumptions can be changed. First, it is possible to fix the

Table 2: Overview of endogenous variables

Description variables	Variable	Dimension
Demand		
Quantity of category c goods in j	q_j^c	$3J$
Quantity bought by the group of agents ag in sector s in j	q_{js}^{ag}	$3JS + JS^2$
Price of utility in j	p_j^u	J
Price of category c goods in j	p_j^c	$3J$
Price of goods bought by ag in sector s in j	p_{js}^{ag}	$3JS + JS^2$
Production		
Quantity of value added in sector s in i	q_{is}^{va}	JS
Quantity of endowment e in sector s in i	q_{ise}^{end}	JSE
Quantity of endowment e in i	q_{ie}^{end}	JE
Price of gross output in sector s in i	p_{is}^{ib}	JS
Price of value added in sector s in i	p_{is}^{va}	JS
Price of endowment e in i	w_{ie}	JE
Price of endowment e in sector s in i	ω_{ise}	JSE
Savings and investment		
Quantity of beginning-of-period capital in i	kb_i	J
Quantity of end-of-period capital in i	ke_i	J
Quantity of investment in i	q_i^{in}	J
Current real rate of return on investment in i	r_i	J
Expected rate of return on investment	r^e	1
Price of investment in i	p_i^{in}	J
Share of investment in country i in global investment	χ_i	J
International trade		
Quantity of goods in sector s in j from source $so = d, m$	q_{js}^{so}	$2JS$
Quantity of goods imported from i by j in sector s	q_{ijs}	J^2S
Price of goods in sector s in j sourced from $so = d, m$	p_{js}^{so}	$2JS$
Price of goods imported from i by j in sector s	p_{ijs}	J^2S
Transport services		
Quantity of transport services demanded on the route from i to j in sector s	ts_{ijs}	J^2S
Global quantity of transport services demanded	ts	1
Quantity of transport services demanded from country i	ts_i	J
Price of transport services demanded on the route from i to j in sector s	p_{ijs}^{ts}	J^2S
Price of global transport services	p^{ts}	1
Price of transport services demanded from i	p_i^{ts}	J
Goods market equilibrium and income		
Quantity of gross output in sector s in i	q_{is}^{ib}	JS
Sum of indirect tax revenues	tr_i^{ind}	J
Expenditure in j	x_j	J
Gross factor income	y_i	J
Tax revenues		
Tax on use of endowment e in sector s	tnr_{ise}	JSE
Direct income tax revenues on endowment e	tdr_{ie}	JE
Tax revenues on gross output (production) in sector s	tpr_{is}	JS
Tax revenues on purchases by group ag from source so	$tar_{js}^{so,ag}$	$6JS$
Tariff revenues on imports from i to j in sector s	tar_{ijs}	J^2S
Tax revenues on exports from i to j in sector s	ter_{ijs}	J^2S

Table 3: Overview of equilibrium equations

Equations	Equation nr.	Dimension	Compl. Variable
Demand			
Deman for category c goods in j	(2)	$3J$	q_j^c
Demand for private and government goods in sector s in j	(4)	$2JS$	$q_{js}^{ag}; ag = pr, go$
Price of utility in j	(3)	J	p_j^u
Price of private and government goods in j	(5)	$2J$	$p_j^c; c = pr, go$
Price of goods bought by ag in sector s in j	(27)	$3JS + JS^2$	$p_{js}^{ag}; ag = pr, go, in, fi_s$
Production			
Demand for value added in sector s in i	(13)	JS	q_{is}^{va}
Demand for sector r intermediates from sector s in i	(14)	JS^2	$q_{ir}^{fi_s}$
Price of gross output in sector s in i	(15)	JS	p_{is}^{ib}
Price of value added in sector s in i	(16)	JS	p_{is}^{va}
Demand for production factor e in sector s in i	(17)	JSE	q_{ise}^{end}
Factor price of (mobile) capital in sector s	(18)	JSE	ω_{isca}
Factor market equilibrium of (mobile) capital in i	(19)	J	w_{ica}
Supply of immobile production factors e to sector s in i	(21)	$5JS$	ω_{ise}
Price of immobile production factor e in i	(22)	$5J$	w_{ie}
Supply of exogenous production factors e in i	exogenous	JE	q_{ie}^{end}
Savings and investment			
Beginning-of-period capital stock in i equal to exogenous capital stock	(32)	J	kb_i
Demand for investment in i following from difference between beginning- and end-of-period capital stock	(31)	J	q_i^{in}
Demand for investment goods in sector s in i	(34)	JS	q_{is}^{in}
Price of investment goods in i	(35)	J	p_i^{in}
Current real rate of return in i	(36)	J	r_i
Expected rate of return as a function of current rate of return and change in capital stock	(37)	J	ke_i
Price of savings in country i	(38)	J	p_i^{sa}
Share of investment in country i in global investment	(39)	J	χ_i
Savings equal to investment	(40)	1	redundant
International trade			
Demand for goods in sector s in j from source $so = d, m$	(24)	JS	q_{js}^{so}
Demand for sector s goods from i in j	(26)	J^2S	q_{ijs}
Price of domestic goods in sector s in j	(28)	JS	p_{js}^d
Price of imported goods in sector s in j	(29)	JS	p_{js}^m
Price of sector s goods from i in j	(30)	J^2S	p_{ijs}
Transport services			
Demand for transport services from i to j in sector s	(43)	J^2S	ts_{ijs}
Global demand for transport services	(45)	1	ts
Price of transport services from i to j in sector s	(44)	J^2S	p_{ijs}^{ts}
Global price of transport services	(46)	1	p^{ts}
Supply of transport services from i	(47)	JM	ts_i
Goods market equilibrium and income			
Goods market equilibrium in sector s in i	(55)	JS	q_{is}^{ib}
Total per capita expenditure in j	(59)	J	x_j
Equation for gross income	(60)	J	y_j
Sum of indirect tax revenues	(54)	J	tir_i
Tax revenues			
Tax revenues on use of endowment e in sector s	(48)	JSE	tnr_{ise}
Income tax revenues on endowment e	(49)	JE	tdr_{ie}
Tax revenues on gross output in sector s	(50)	JS	tpr_{is}
Tax revenues on purchases by group ag from source so	(51)	$6JS$	$tar_{js}^{so,ag}$
Tariff revenues on imports from i to j in sector s	(52)	J^2S	tar_{ijs}
Tax revenues on exports from i to j in sector s	(53)	J^2S	ter_{ijs}

trade balance. As this would make an endogenous variable exogenous, this makes it necessary to endogenize another variable. To see which variable could be endogenized, we look at the international macroeconomic identity imposing that the surplus of investment over savings plus the surplus of tax revenues over government expenditures is equal to the trade surplus, so the difference between exports and imports:

$$(S - I) + (T - G) = (X - M) \tag{62}$$

Savings S is a fixed share of household income given the Cobb-Douglas specification for spending across different types of goods. Investment I is determined by the global bank based on the attractiveness to invest in a region. Tax reveues T are determined by the tax base and fixed tax rates and government expenditures G are fixed share of household income given the Cobb-Douglas specification. Hence, the trade balance $X - M$ varies in the model based on the four "macro" variables described and fixing the trade balance without changing the specification of one of the four variables woul overidentify the model. Fixing the trade balance is typically implemented in two different ways. The share of income spent on savings could be endogenized, such that this share adjusts to satisfy equation (62) with a fixed trade balance. Or the equation equalizing the expected rate of return across regions can be switched off such that investments can adjust to satisfy equation (62) with a fixed trade balance. To preserve homogeneity in nominal variables, it would be best to fix the trade balance ratio, such that a change in all prices such as a numeraire shock does not have real effects.

Second, employment can be endogenized. The easiest and most reduced-form way to do so is by fixing the real wage and endogenizing the real wage. This closure reflects a short-run perspective with rigid real wages. As an alternative the model could be extended with endogenous labour supply with an upward-sloping labour-supply curve.

Other so-called closure swaps would be to fix the budget deficit and adjust the expression for tax revenues or government expenditures or to fix the exchange rate. However, both options are not available in the standard GTAP model, since there is no expression for the budget deficit and since there are no explicit exchange rates in the model. The real bilateral exchange rate between two countries is defined implicitly as for example the ratio of factor prices between two countries.

3 Concluding Remarks

This paper has given an overview of the formal details of the most well-known CGE model on international trade, GTAP. In the overview the differences with the new quantitative trade (NQT) models have been identified, consisting of the presence savings and investment, substitution elasticities different from 1 across the different nests, different import shares by end user, non-homothetic preferences, and the presence of export subsidies, other taxes, and a separate transport sector. In general these differences can be explained from the following differences in philosophy across the two literatures. First, CGE modelers tend to use parameters from the literature if they seem reasonable, whereas NQT practitioners attempt to be rigorous on structural estimation, thus not borrowing parameters from the literature. This can explain the fact that NQT models work with lots of Cobb-Douglas nests and thus without explicit parameter values on substitution elasticities. However, it can also explain the omission of savings, investment and capital in almost all NQT models. In the CGE-model discussed savings and investment are modelled with a global bank, featuring parameters that cannot be estimated. Second, CGE models attempt to stay close to real-world data and include many institutional details such as different types of (international) taxes, whereas NQT models prefer parsiminious models in order to prevent that the model becomes too complicated and difficult to interpret. In this paper no value judgement is given, it is just observed that NQT models could be nested as special cases of CGE models by stripping the model of savings, investment, and capital, eliminating different types of taxes and the transport sector, and collapsing various choice nests to Cobb-Douglas.

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Appendix A CES Preferences

Utility of a consumer, q, is a function of the quantity of good i consumed, q_i:

$$q = \left(\sum_{i=1}^{n} \alpha_i q_i^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}}$$

σ is the substitution elasticity and α_i is a taste shifter. The demand function corresponding to the utility function is given by the following expression with y consumer income:

$$q_j = \frac{\alpha_j^{\sigma} p_j^{-\sigma}}{\sum_{i=1}^{J} \alpha_i^{\sigma} p_i^{1-\sigma}} y \tag{A.1}$$

We can convert Marshallian (indirect) demand (as a function of prices and income) into Hicksian (indirect) demand (as a function of prices and utility) as follows with income y equal to the price index times utility, pq:

$$q_j = \left(\frac{\alpha_j p}{p_j} \right)^{\sigma} q \tag{A.2}$$

p is the price index, an average over all prices, and formally defined as the price of one unit of utility, q:

$$p = \left(\sum_{i=1}^{J} \alpha_i^{\sigma} p_i^{1-\sigma} \right)^{\frac{1}{1-\sigma}} \tag{A.3}$$

The expressions for demand in equations (A.1)-(A.2) is intuitive. First, an increase in the own price p_j relative to average price p reduces demand q_j. Second, demand q_j for each good is proportional to utility q and income y. This reflects that CES-preferences are homothetic.

In our CGE-model all demand functions except for private household demand across sectors are defined as CES-functions. The value of σ is crucial for the response of budget shares to changes in prices. The budget share, s_i, the share of income spent on good i, is defined as:

$$s_j = \frac{p_j q_j}{y} = \frac{\alpha_j^{\sigma} p_j^{1-\sigma}}{\sum_{i=1}^{J} \alpha_i^{\sigma} p_i^{1-\sigma}} \tag{A.4}$$

Hence the value of σ determines the response of the budget share, s_j, to a change in the price of good j, p_j. In particular, the budget share is:

1. Increasing in p_j when $\sigma < 1$: goods are gross complements

2. Constant in p_j when $\sigma = 1$

3. Decreasing in p_j when $\sigma > 1$: goods are gross substitutes

Two special cases can be identified:

1. $\sigma = 1$: Cobb-Douglas preferences: budget shares are constant in prices (price and volume effects cancel out)

2. $\sigma = 0$: Leontief preferences: the budget share of good i is rising in the price of good i (only price effect, no volume effect); quantity q_i is proportional to utility q:

$$q_i = q \tag{A.5}$$

Appendix B Derivations Demand

Log differentiating the expenditure function in equation (10) with respect to utility q_j^{pr}, prices p_{js}^{pr}, and expenditure x_j^{pr} and solving for $\widehat{x_j^{pr}}$ leads to:

$$\widehat{x_j^{pr}} = \sum_{s=1}^{S} \frac{\alpha_s \left(q_j^{pr}\right)^{\gamma_s \eta_s} \left(\frac{p_{js}^{pr}}{x_j^{pr}}\right)^{\gamma_s} \gamma_s}{\sum_{u=1}^{S} \alpha_u \left(q_j^{pr}\right)^{\gamma_u \eta_u} \left(\frac{p_{ju}^{pr}}{x_j^{pr}}\right)^{\gamma_u} \gamma_u} \left(\widehat{p_{js}^{pr}} + \eta_s \widehat{q_j^{pr}}\right) = \sum_{s=1}^{S} s_{js}^{pr} \widehat{p_{js}^{pr}} + \sum_{s=1}^{S} s_{js}^{pr} \eta_s \widehat{q_j^{pr}} \tag{B.1}$$

Variables with a hat indicate relative changes, i.e. $\widehat{x} = \frac{dx}{x}$. Equation (B.1) can be used to derive two equations in the main text, the expression for sectoral private demand, q_{js}^{pr}, in equation (11) and the expression for the elasticity of quantity, q_j^c, with respect to expenditure, x_j^c, in equation (8).

s_{js}^{pr}, the coefficient on $\widehat{p_{js}^{pr}}$, is the expenditure share on good s by Shepherd's lemma, i.e. $s_{js}^{pr} = \frac{dx_j^{pr}}{dp_{js}^{pr}} \frac{p_{js}^{pr}}{x_j^{pr}} = \frac{q_{js}^{pr} p_{js}^{pr}}{x_j^{pr}}$. Recall that Shepherd's lemma shows that the partial derivative of the expenditure function, x_j^{pr}, with respect to the price of the good from sector s, p_{js}^{pr}, gives Hicksian demand, which can be converted into Marshallian demand by writing utility as a function of expenditure. From equation (B.1) we can thus find the expression for demand q_{js}^{pr} in equation

(11) using the expression for s_{js}^{pr} and the fact that $q_{js}^{pr} = s_{js}^{pr}\frac{x_{js}^{pr}}{p_{js}^{pr}}$:

$$q_{js}^{pr} = s_{js}^{pr}\frac{x_{js}^{pr}}{p_{js}^{pr}} = \frac{\alpha_s\left(q_j^{pr}\right)^{\gamma_s\eta_s}\left(\frac{p_{js}^{pr}}{x_j^{pr}}\right)^{\gamma_s-1}\gamma_s}{\sum\limits_{u=1}^{S}\alpha_u\left(q_j^{pr}\right)^{\gamma_u\eta_u}\left(\frac{p_{ju}^{pr}}{x_j^{pr}}\right)^{\gamma_u}\gamma_u} \tag{B.2}$$

The coefficient on $\widehat{q_j^{pr}}$, $\sum\limits_{s=1}^{S} s_{js}^{pr}\eta_s$, is the inverse of the elasticity of utility with respect to expenditure, Ψ_j^{pr} in equation (8).

To derive equations (7) and (9) we maximise utility in equation (1) subject to the implicit budget constraint in equation (6). The first order conditions (FOCs) are given by:

$$\kappa_j^c\frac{u_j}{q_j^c} = \lambda\frac{\partial e_j^c}{\partial q_j^c} \tag{B.3}$$

λ is the Lagrange multiplier of the maximisation problem. Combining the FOCs, defining Ψ_j^c as the elasticity of quantity with respect to expenditure, $\Psi_j^c = \frac{\partial q_j^c}{\partial e_j^c}\frac{x_j^c}{q_j^c}$, and substituting the result into the budget constraint leads to the following expression for x_j^c:

$$x_j^c = \frac{\kappa_j^c\Psi_j^c}{\sum\limits_{d\in\{pr,go,sa\}}\kappa_j^d\Psi_j^d}x_j \tag{B.4}$$

We get then to equation (7) using the expression for Ψ_j in equation (9). To derive equation (9) we log differentiate the utility function in (1). Applying the definition of Ψ_j^c gives:

$$\widehat{u_j} = \sum_{c\in\{p,g,s\}}\kappa_j^c\widehat{q_j^c} = \sum\kappa_j^c\Psi_j^c\widehat{x_j^c} \tag{B.5}$$

Log differentiating the budget constraint and substituting equation (7) generates:

$$\sum_{c\in\{p,g,s\}}\kappa^c\frac{\Psi_j^c}{\sum\limits_{d\in\{p,g,s\}}\kappa_j^d\Psi_j^d}\widehat{x_j^c} = \widehat{x_j} \tag{B.6}$$

Substituting equation (B.5) into equation (B.6) we get:

$$\frac{1}{\sum\limits_{d\in\{p,g,s\}}\kappa_j^d\Psi_j^d}\widehat{u_j} = \widehat{x_j} \tag{B.7}$$

Hence, equation (B.7) shows that the elasticity of utility u_j with respect to expenditure x_j is given by the expression in equation (9).