New global estimates of import demand elasticities: a technical note

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15 July 2020

Introduction

This study provides new estimates of import demand elasticities for over 180 countries and all traded products at the 2012 harmonized system (HS) six-digit level using state-of-the-art estimation techniques and computing hardware based on trade data up to 2019. Limited Information Maximum Likelihood (LIML), together with nonlinear LIML constrained to fix infeasible estimates, proposed by Soderbery (2015), is used in constructing an initial dataset based on the 2002 HS six-digit classification. A concordance and filling exercise then enables transformation of elasticities to the 2012 HS version for all traded products. Raw data (before the concordance and filling exercise) contains almost 300,000 elasticities of import demand, covering more than 5,000 product lines in 185 economies. The unique computation method employed in this study has only previously been employed for estimations of limited geographical scope due to the large computing power requirements. This study employed advanced processing hardware and efficiency increasing techniques to make it possible to apply this method to estimate a global dataset of elasticities.

Literature review

The main contribution to derivation of import elasticity literature is from Feenstra (1994) and Broda and Weinstein (2006), where the setup is based on microeconomic foundation on constant elasticity of substitution (CES) demand function, which is widely used in international trade models. Feenstra (1994) introduces product varieties to import price indices. The setup allows the change in varieties and quality of product over time. Broda and Weinstein (2006) contributes to the more accurate estimation by applying constrained grid search to infeasible estimates of elasticities of import demand. Soderbery (2010, 2015) shows that estimates from both Feenstra (1995) and Broda and Weinstein (2006) could suffer from small sample bias, especially when time
panel is low. Soderbery (2015) applies LIML to estimate elasticities of import demand, with application of constrained LIML search if first stage estimates are infeasible. The study showed results from methods potentially overestimate elasticities by 35%.

The main advantage of the Soderbery (2015) model is the simple data requirement in estimating elasticities of substitution and in turn elasticities of import demand. Import prices and shares of each product can be acquired from values and quantities of imports. Only one trade source of data is needed. However, most studies that use Soderbery (2015) in the literature have limited their scope by either geographical location, coverage of goods or time period due to computational requirements.

Benkovskis and Wörz (2014) estimates elasticities based on Soderbery (2010), which is the basis of methodology in Soderbery (2015), and scope their study on 4 European G7 countries, namely, France, Germany, Italy and the United Kingdom with 50 trading partners between 1995 and 2012 on product at 8-digit Combined Nomenclature (CN) classification. Median of the result of elasticities of substitution between varieties in all 4 countries are all closed to 3, with maximum greater than 90000. Benkovskis and Wörz (2018) extends their work to 188 countries between 1996 and 2011 on product at 6-digit HS1996 classification. Median of the result of elasticities of substitution between varieties in its top 10 countries are not exceeding 3.8, with maximum greater than 950000. Kancs and Persyn (2019) scopes their study in Baltic countries, namely, Estonia, Latvia and Lithuania during 1988-1997. More than 80% of their elasticities of import demand are not exceeding 2. Brucal and Roberts (2019) focuses their work specifically on home appliance in the United States.

Another alternative method is based on Arlington Model or structural gravity model, where trade costs are incorporated in the model setup. However, most of these models need trade cost proxies, for example, distance or geographical/border information, for trade cost model to retrieve elasticity of substitution, as in Anderson and van Wincoop (2003) and Bergstrand et al. (2013). Other proxies are also discussed in the literature. Ad valorem bilateral tariff rates and cif/fob ratios are also discussed as alternatives in measuring trade costs as it is considered as the direct price shifter. However, most gravity models do not use such measures because they are subject to measurement error. Tariff issues are extensively mentioned in Anderson and van Wincoop (2004) and Hummels

Kee et al. (2008), and the updated import demand elasticities from Ghodsi et al. (2016), propose systematically deriving elasticities of import demand from the application of semi-flexible translog GDP function approach to estimate elasticities of import demand from prices and factor endowment. However, infeasible elasticities of import demand in this study are still accounted for 5% of the estimates and are excluded from the model. Apart from the infeasible elasticities, more explanatory variables on factor endowments which could be missing in many developing countries, may result in possibility of missing estimates when prices and shares of import are available but factor endowments are not. Indeed, Kee et al. (2008) calculate elasticities for only 149 economies. Resembling Kee et al. (2008), Chen and Novy (2011) follow derivation of elasticity of substitution based on literature from Imbs and Méjean (2009) where domestic output, import shares and prices are used. As data in industrial classification may not exactly match trade nomenclature, analysis in most disaggregated level may not possibly be accomplished.

This study thus, uses the latest trade data (up to 2019) with methodology that, although computational tasking, and results in inclusion of traditionally overlooked economies in estimation of demand elasticities.

**Methodology**

This study applies Soderbery (2015) methodology in estimating elasticities of import demand. Let a consumer have nested CES preference on domestic and imported goods and their varieties. Let \( v \) a set of varieties, which is defined by export origin of such importation, of good \( g \) at time \( t \), denoted by \( I_{gt} \subset \{1, 2, \ldots, v, V\} \). Then, consumption of a good \( g \) variety \( v \) at time \( t \) is denoted as \( x_{gvt} \). Denote \( \sigma_g > 1 \) be constant elasticity of substitution for a good \( g \) and denote change in taste as \( b_{gvt} \). Thus, utility function for variety of good \( g \) is given by

\[
X_{gt} = \left( \sum_{v \in I_{gt}} \frac{1}{B_{gvt}} \frac{\sigma_g}{\sigma_g - 1} x_{gvt}^{\frac{\sigma_g}{\sigma_g - 1}} \right)^{\frac{\sigma_g}{\sigma_g - 1}}
\]

Demand of a variety \( v \) good \( g \) at time \( t \) is
\[ x_{gvt} = \left( \frac{p_{gvt}}{\sum p_{gvt} b_{gvt}} \right)^{1-\sigma_g} \cdot \frac{1}{p_{gvt}} \cdot E_{gt} b_{gvt} \text{ where price index } P_{gt} = \left( \sum_{g \in I_{gt}} p_{gvt} b_{gvt} p_{gvt}^{1-\sigma_g} \right)^{1-\sigma_g}. \]

Thus, market share is

\[ s_{gvt} \equiv \frac{p_{gvt} x_{gvt}}{\sum_{v \in I_{gt}} p_{gvt} x_{gvt}} = \left( \frac{p_{gvt}}{\sum_{g \in I_{gt}} p_{gvt}} \right)^{1-\sigma_g} b_{gvt} \quad (2) \]

Therefore, market share is determined by its price \((p_{gvt})\) relative to price index and variety preference \((b_{gvt})\).

On the supply side, exporters face monopolistic competitive market in the following form:

\[ p_{gvt} = \left( \frac{\sigma_g}{\sigma_g - 1} \right) \exp (\eta_{gvt}) (x_{gvt})^\omega_g \quad (3) \]

where \(\omega_g \geq 0\) denotes elasticity of export supply and \(\eta_{gvt}\) denotes random technology factor on supply side. By re-arranging terms on supply equation in terms of market share, it can be written as

\[ p_{gvt} = \left( \sum_{v \in I_{gt}} \exp \left( \frac{-\eta_{gvt}}{\omega_g} \right)^{1+\omega_g} p_{gvt}^{\omega_g} \right)^{\omega_g} \cdot \exp \left( \frac{\eta_{gvt}}{1+\omega_g} \right)^{\omega_g} s_{gvt}^{\omega_g} \quad (4) \]

Feenstra (1995) removes the time and product specific effect by first differencing prices and shares, where \(\Delta\) denotes first difference, and then time specific effect, where \(\Delta^k\) denotes first difference with reference country \(k\). Then, import demand equation and export supply of good \(g\) can be expressed in the following form:
\[ \Delta^k \ln s_{gvt} \equiv \Delta \ln (s_{gvt}) - \Delta \ln (s_{gkt}) = -(\sigma_g - 1) \Delta^k \ln (p_{gvt}) + \varepsilon^k_{gvt} \quad (5) \]

\[ \Delta^k \ln p_{gvt} \equiv \Delta \ln (p_{gvt}) - \Delta \ln (p_{gkt}) = \left( \frac{\omega_g}{1+\omega_g} \right) \Delta^k \ln (s_{gvt}) + \delta^k_{gvt} \quad (6) \]

where \( \varepsilon^k_{gvt} = \Delta^k \ln (b_{gvt}) \) denotes unobservable random demand shock and \( \delta^k_{gvt} = \Delta^k \left( \frac{\eta_{gvt}}{1+\omega_g} \right) \)

denotes unobservable random production shock. Multiplying (5) and (6) and re-arranging terms, the study shows the estimation can be executed with 1 equation as follows:

\[ Y_{gvt} = \theta_{1g} X_{1gvt} + \theta_{2g} X_{2gvt} + u_{gvt} \quad (7) \]

where

\[ Y_{gvt} \equiv (\Delta^k \ln (p_{gvt}))^2 \]

\[ X_{1gvt} \equiv (\Delta^k \ln (s_{gvt}))^2 \]

\[ X_{2gvt} \equiv (\Delta^k \ln (p_{gvt}))(\Delta^k \ln (s_{gvt})) \] and

\[ u_{gvt} = \frac{\varepsilon^k_{gvt} \delta^k_{gvt}}{\sigma_g - 1} \quad \text{where} \quad \rho_g = \frac{\omega_g (\sigma_g^{-1})}{1-\omega_g \sigma_g} \in \left[ 0, \frac{\sigma_g^{-1}}{\sigma_g} \right] \]

\[ \theta_{1g} \equiv \frac{\rho_g}{(\sigma_g - 1)(1-\rho_g)} \] and \( \theta_{2g} \equiv \frac{2 \rho_g^{-1}}{(\sigma_g - 1)(1-\rho_g)} \)

Equation (7), together with the coefficient \( \theta_{1g} \) and \( \theta_{2g} \) will solve for both import demand and export supply function simultaneously. Soderbery (2015) proposes the use of Limited Information Maximum Likelihood (LIML), together with nonlinear LIML constrained to fix infeasible estimates, in estimating elasticities of import demand in order to avoid the estimation bias due to small sample sizes in some product lines and obtain the feasible estimates.
Results

In terms of data acquisition, the study obtains import value and quantity from the Commodity Trade Statistics Database (COMTRADE), which is obtained through World Integrated Trade Solution (WITS) platform. Data covers 2002-2019 and 2012-2019 for 6-digit HS2002. Raw dataset from HS2002 covers 185 economies and 5156 product lines, while HS2012 covers 147 reporting countries, with 5,095 product line coverage. Due to the exponential nature of estimation, some values were inevitably indefensibly large. Approximately 3.5% of elasticities had a magnitude of larger than 100 (see figure 1). Indeed, Soderbery (2015) and other studies using this methodology noted the issue of outliers. While using a longer time period (discussed below) could have addressed some of them, as discussed below, it was decided that “freshness” of elasticities in the context of derived demand based on current trade flows was considered to be a more important attribute of the dataset. As such, following Kee et al. (2008), the 5% of elasticities with highest magnitude were removed.

Figure 1. Frequency distribution of import demand elasticities with magnitude less than 100

Table 1 and figure 2 provide some elements of comparison between the global dataset of elasticities by Kee et al. (2008)³ and this new dataset. The new dataset covers a larger number of

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³ The original paper Kee et al (2008) notes that elasticities were derived for 117 economies, with estimation using trade data from 1988-2002. However, the World Bank dataset repository (see https://datacatalog.worldbank.org/dataset/overall-trade-restrictiveness-indices-and-import-demand-elasticities) suggests the dataset was last updated in 2012, but no additional technical information is provided. As such, the
products as well as economies, including significantly larger number of developing economies, although the number of derived unique elasticities in the new dataset is lower than in Kee et al. (2008). Furthermore, this study used updated trade data and limited the starting time period for estimation from 2002 to 2018 (14 years), whereas the original Kee et al. (2008) seems to have used an earlier starting point as well as longer time period. While longer time frame offers more degrees of freedom to increase the pool of estimated elasticities, using older trade flows also raises the question of relevance of the information derived to today’s modern economy. Lastly, the new dataset provides clarity in terms of which elasticities apply to the changing HS code versions over time.⁴

<table>
<thead>
<tr>
<th></th>
<th>This study</th>
<th>Kee et al. (2008)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of unique reporter-specific elasticities⁵</td>
<td>298,612</td>
<td>377,616</td>
</tr>
<tr>
<td>Number of economies</td>
<td>185</td>
<td>149</td>
</tr>
<tr>
<td>Number of six-digit HS codes</td>
<td>5,156 (HS2002)</td>
<td>4,900 (mixed HS versions)</td>
</tr>
<tr>
<td>Time span used</td>
<td>2002-2018</td>
<td>1988-2002⁶</td>
</tr>
</tbody>
</table>

⁴ Mixed HS codes in the Kee et al. (2008) results suggest that various HS codes may have been lumped together in estimation, raising question on the accuracy of the elasticities estimated on the basis of inconsistent code versions.
⁵ This is before the concordance and filling exercises are conducted.
⁶ The original paper Kee et al (2008) notes that elasticities were derived for 117 economies, with estimation using trade data from 1988-2002. However, the World Bank dataset repository suggests the dataset was last updated in 2012 (and numbers match to what we find in SMART), but no additional technical information is provided. HS code versions in the latest dataset are mixed.
Notably, not every of the 185 economies used had a full set of elasticities for each of the 5,156 product codes, in most cases due to small sample size. In such cases, the global averages for each HS code was used. Finally, the study replaces missing data in HS2012 with HS2002 using concordance data from WITS via simple average function.

7 This also seems to be the strategy employed when importing demand elasticities into WITS SMART analysis tool to ensure simulations are run for all countries.
References


Annex: Derivation of estimated equation for import demand and export supply

Demand function:

Assume the representative consumer of country i (which is omitted in the equation) faces constant elasticity of substitution (CES) demand function for both domestic and foreign goods. Let variety $v$ of good $g$ at time $t$ denoted by $I_{gt} \subset \{1, 2, \ldots, v, V\}$. Total quantity of each variety of good $g$ consumed in period $t$ is denoted by $x_{gvt}$ and $\sigma_g > 1$ denotes the CES of good $g$. Let $b_{gvt}$ be an exogenous taste shifter. Let us focus on the demand on imported goods. Then, utility function is given by

$$U_{gt} = \left( \sum_{v \in I_{gt}} \frac{1}{\sigma_g} x_{gvt} \frac{\sigma_g}{\sigma_g - 1} \right)^{\frac{1}{\sigma_g - 1}}$$ (1)

Then, utility maximizing problem is given by

$$\max_{x_{gvt}} \left( \sum_{v \in I_{gt}} \frac{1}{\sigma_g} x_{gvt} \frac{\sigma_g}{\sigma_g - 1} \right)^{\frac{1}{\sigma_g - 1}} \text{ subject to } \sum_{v \in I_{gt}} p_{gvt} x_{gvt} \leq X_{gt}$$

Then, Lagrangian function $(L)$ is:

$$L: \left( \sum_{v \in I_{gt}} \frac{1}{\sigma_g} x_{gvt} \frac{\sigma_g}{\sigma_g - 1} \right)^{\frac{1}{\sigma_g - 1}} - \lambda \left( \sum_{v \in I_{gt}} p_{gvt} x_{gvt} - X_{gt} \right)$$

First-order conditions are given by

$$\frac{\partial L}{\partial x_{gvt}} = 0 \iff \left( \frac{1}{\sigma_g} \right) \left( \sum_{v \in I_{gt}} \frac{1}{\sigma_g} x_{gvt} \frac{\sigma_g}{\sigma_g - 1} \right)^{\frac{1}{\sigma_g - 1}} \frac{1}{\sigma_g} \frac{1}{\sigma_g} \frac{1}{\sigma_g} = \lambda p_{gvt}$$ (2)
Thus,

\[
\left( \sum_{v \in g_t} b_{gvt}^\sigma x_{gvt} \right) \frac{1}{\sigma g - 1} \frac{1}{\sigma g - 1} b_{gvt}^\sigma x_{gvt} = \lambda p_{gvt} \tag{4}
\]

From (4), let us consider a good g with 2 varieties v and v’, then

For variety v:

\[
b_{gvt}^\sigma g \cdot x_{gvt} = \lambda p_{gvt} \tag{5}
\]

For variety v’:

\[
b_{gvt}^\sigma g \cdot x_{gvt} = \lambda p_{gvt} \tag{6}
\]

Dividing (5) by (6),

\[
\frac{b_{gvt}}{b_{gvt}^\sigma g} = \frac{p_{gvt}^\sigma g}{p_{gvt}^\sigma g} x_{gvt} \cdot x_{gvt} \tag{7}
\]

Multiplying both sides of (7) with \(p_{gvt}\) and re-arranging terms

\[
p_{gvt} x_{gvt} = \frac{p_{gvt}^\sigma g}{b_{gvt}} x_{gvt} \cdot b_{gvt}^\sigma g \cdot b_{gvt} \cdot p_{gvt}^{1 - \sigma g}
\]

Taking summation over \(v\), thus
\[ \sum_{\nu \in \mathcal{V}_t} p_{g\nu t} x_{g\nu t} = \frac{p_{g\nu t}^{\sigma_g}}{b_{g\nu t}} \sum_{\nu \in \mathcal{V}_t} b_{g\nu t} p_{g\nu t}^{1-\sigma_g} \]

Let \( \left( \sum_{\nu \in \mathcal{V}_t} b_{g\nu t} p_{g\nu t}^{1-\sigma_g} \right)^{\frac{1}{1-\sigma_g}} = p_{gt} \). Thus, \( \sum_{\nu \in \mathcal{V}_t} b_{g\nu t} p_{g\nu t}^{1-\sigma_g} = p_{gt}^{1-\sigma_g} \). Denote \( E_{gt} = \sum_{\nu \in \mathcal{V}_t} p_{g\nu t} x_{g\nu t} \), then

\[ E_{gt} = \frac{p_{g\nu t}^{\sigma_g}}{b_{g\nu t}} x_{g\nu t} \cdot p_{gt}^{1-\sigma_g} \]

Re-arranging terms to have import demand as a function of price:

\[ x_{g\nu t} = \left( \frac{p_{g\nu t}}{p_{gt}} \right)^{1-\sigma_g} \cdot \frac{1}{p_{g\nu t}} \cdot E_{gt} b_{g\nu t} \tag{8} \]

Define market share \( s_{g\nu t} = \frac{p_{g\nu t} x_{g\nu t}}{\sum_{\nu \in \mathcal{V}_t} p_{g\nu t} x_{g\nu t}} = \frac{p_{g\nu t} x_{g\nu t}}{E_{gt}}. \) Therefore,

\[ s_{g\nu t} = \frac{p_{g\nu t}}{E_{gt}} \cdot \frac{1}{p_{g\nu t}} \cdot p_{g\nu t} E_{gt} b_{g\nu t} = \left( \frac{p_{g\nu t}}{p_{gt}} \right)^{1-\sigma_g} b_{g\nu t} \tag{9} \]

Taking \( \ln \) on both side of the equation, we have

\[ \ln s_{g\nu t} = -(\sigma_g - 1) \ln p_{g\nu t} + (\sigma_g - 1) \ln p_{gt} + \ln b_{g\nu t} \tag{10} \]

**Supply function:**

Let \( \eta_{g\nu t} \) be a technological shifter in production function where \( \omega_g \geq 0 \) denotes supply elasticities. Exporters in monopolistic competitive market facing upward sloping export supply as follows:
\[ p_{gvt} = \left( \frac{\sigma_g}{\sigma_g - 1} \right) \exp (\eta_{gvt}) \left( x_{gvt} \right)^{\omega_g} \]  

(11)

Re-arranging terms for \( x_{gvt} \), so we have

\[ x_{gvt} = \left[ \left( \frac{\sigma_g}{\sigma_g - 1} \right) p_{gvt} \exp (-\eta_{gvt}) \right]^\frac{1}{\omega_g} \]

Multiplying both sides with \( p_{gvt} \), thus

\[ p_{gvt} x_{gvt} = \left( \frac{\sigma_g - 1}{\sigma_g} \right)^{\frac{1}{\omega_g}} \left( p_{gvt} \right)^{\frac{1+\omega_g}{\omega_g}} \exp \left( \frac{-\eta_{gvt}}{\omega_g} \right) \]  

(12)

Re-arranging terms in form of share, we get

\[ s_{gvt} = \frac{p_{gvt} x_{gvt}}{\sum_{vel_{gt}} p_{gvt} x_{gvt}} = \frac{\left( \frac{1+\omega_g}{\omega_g} \right) p_{gvt} \exp \left( \frac{-\eta_{gvt}}{\omega_g} \right)}{\sum_{vel_{gt}} \left( \frac{1+\omega_g}{\omega_g} \right) p_{gvt} \exp \left( \frac{-\eta_{gvt}}{\omega_g} \right)} \]

Re-arranging equation to get supply equation, \( p_{gvt} \), then

\[ p_{gvt} = s_{gvt}^{\frac{\omega_g}{1+\omega_g}} \exp \left( \frac{\eta_{gvt}}{1+\omega_g} \right) \left\{ \sum_{vel_{gt}} \left[ \frac{\omega_g}{1+\omega_g} \right] p_{gvt} \exp \left( \frac{-\eta_{gvt}}{\omega_g} \right) \right\}^{\frac{\omega_g}{1+\omega_g}} \]  

(13)

Taking \( \ln \) on both side of the equation, we have

\[ \ln p_{gvt} = \left( \frac{\omega_g}{1+\omega_g} \right) \ln s_{gvt} + \left( \frac{\eta_{gvt}}{1+\omega_g} \right) + \left( \frac{\omega_g}{1+\omega_g} \right) \ln \left\{ \sum_{vel_{gt}} \left[ \frac{\omega_g}{1+\omega_g} \right] p_{gvt} \exp \left( \frac{-\eta_{gvt}}{\omega_g} \right) \right\} \]  

(14)
To eliminate good-specific unobservables, first differencing (denoted by $\Delta$) share equation (10) and price equation (14)

$$\Delta \ln s_{gvt} = - (\sigma_g - 1) \Delta \ln p_{gvt} + \varphi_{gt} + \epsilon_{gvt} \tag{15}$$

where $\varphi_{gt} = (\sigma_g - 1) \Delta \ln p_{gvt}$ and $\epsilon_{gvt} = \Delta \ln b_{gvt}$ are time-product specific unobservables and variety-specific unobservables, respectively, and

$$\Delta \ln p_{gvt} = \left(\frac{\omega_g}{1+\omega_g}\right) \ln s_{gvt} + \psi_{gt} + \delta_{gvt} \tag{16}$$

where $\psi_{gt} = \left(\frac{\omega_g}{1+\omega_g}\right) \ln \left\{ \sum_{v, t} p_{gvt} \frac{\omega_g}{\omega_g} \exp \left(\frac{-\eta_{gvt}}{\omega_g}\right) \right\}$ and $\delta_{gvt} = \left(\frac{\eta_{gvt}}{1+\omega_g}\right)$ are time-product specific unobservables and variety-specific unobservables.

To eliminate time-specific unobservables, the model is differencing with reference country $k$, denoted as $\Delta^k$. Therefore, we have system of equations for elasticities of import demand and export supply as follows:

$$\Delta^k \ln s_{gvt} = \Delta \ln s_{gvt} - \Delta \ln s_{gkt} = - (\sigma_g - 1) \Delta^k \ln p_{gvt} + \epsilon^k_{gvt} \tag{17: Demand Equation}$$

$$\Delta^k \ln p_{gvt} = \Delta \ln p_{gvt} - \Delta \ln p_{gkt} = \left(\frac{\omega_g}{1+\omega_g}\right) \Delta^k \ln s_{gvt} + \delta^k_{gvt} \tag{18: Supply Equation}$$

where $\epsilon^k_{gvt} = \Delta^k \ln b_{gvt}$ and $\delta^k_{gvt} = \Delta^k \left(\frac{\eta_{gvt}}{1+\omega_g}\right)$.

Feenstra (1995) combines these structural demand and supply in (17) and (18) into 1 equation by multiplying $\epsilon^k_{gvt}$ and $\delta^k_{gvt}$ together. Re-arranging (17) and (18), we have

$$\epsilon^k_{gvt} = \Delta^k \ln s_{gvt} + (\sigma_g - 1) \Delta^k \ln p_{gvt} \tag{19}$$

$$\delta^k_{gvt} = \Delta^k \ln p_{gvt} - \left(\frac{\omega_g}{1+\omega_g}\right) \Delta^k \ln s_{gvt} \tag{20}$$
Multiplying (19) and (20), then

\[
\varepsilon_g^k \delta_{gvt}^k = \Delta^k \ln s_{gvt} \Delta^k \ln p_{gvt} + (\sigma_g - 1) \left( \Delta^k \ln p_{gvt} \right)^2 - \left( \frac{\omega_g}{1+\omega_g} \right) \left( \Delta^k \ln s_{gvt} \right)^2 - \\
\left( \frac{\omega_g(\sigma_g-1)}{1+\omega_g} \right) \left( \Delta^k \ln s_{gvt} \Delta^k \ln p_{gvt} \right)
\]

(21)

Let \( \rho_g \equiv \frac{\omega_g(\sigma_g-1)}{1+\omega_g} \in \left[ 0, \frac{\sigma_g-1}{\sigma_g} \right) \). Thus \( 1 - \rho_g = \frac{1+\omega_g}{1+\omega_g \sigma_g} \) and \( \frac{1}{1-\rho_g} = \frac{1+\omega_g \sigma_g}{1+\omega_g} \). We write \( \frac{\omega_g}{1+\omega_g} \) in terms of \( \rho_g \) and \( \sigma_g \) as follows:

\[
\rho_g = \frac{\omega_g(\sigma_g-1)}{1-\omega_g \sigma_g}
\]

Multiplying both sides by \( \left( \frac{1}{1+\omega_g} \right) \)

\[
\frac{\rho_g}{1+\omega_g} = \frac{\omega_g(\sigma_g-1)}{(1+\omega_g)(1+\omega_g \sigma_g)}
\]

\[
\frac{\omega_g}{1+\omega_g} = \frac{\rho_g}{\sigma_g-1} \cdot \frac{1+\omega_g \sigma_g}{1+\omega_g}
\]

\[
\frac{\omega_g}{1+\omega_g} = \frac{1}{\sigma_g-1} \cdot \frac{\rho_g}{1-\rho_g}
\]

(22)

Substituting (22) into (21)

\[
\varepsilon_g^k \delta_{gvt}^k = \Delta^k \ln s_{gvt} \Delta^k \ln p_{gvt} + (\sigma_g - 1) \left( \Delta^k \ln p_{gvt} \right)^2 - \left( \frac{1}{\sigma_g-1} \cdot \frac{\rho_g}{1-\rho_g} \right) \left( \Delta^k \ln s_{gvt} \right)^2 - \\
\left( \frac{\rho_g}{1-\rho_g} \right) \left( \Delta^k \ln s_{gvt} \Delta^k \ln p_{gvt} \right)
\]
\[ \varepsilon_{gvt}^k \delta_{gvt}^k = \left( \frac{1-2\rho_g}{1-\rho_g} \right) \left( \Delta^k \ln s_{gvt} \Delta^k \ln p_{gvt} \right) + (\sigma_g - 1) \left( \Delta^k \ln p_{gvt} \right)^2 - \left( \frac{\rho_g}{\sigma_g-1} \cdot \frac{\rho_g}{1-\rho_g} \right) \left( \Delta^k \ln s_{gvt} \right)^2 \]

\[ (\sigma_g - 1) \left( \Delta^k \ln p_{gvt} \right)^2 = \left( \frac{1}{\sigma_g-1} \cdot \frac{\rho_g}{1-\rho_g} \right) \left( \Delta^k \ln s_{gvt} \right)^2 + \left( \frac{2\rho_g-1}{(\sigma_g-1)(1-\rho_g)} \right) \left( \Delta^k \ln s_{gvt} \Delta^k \ln p_{gvt} \right) + \varepsilon_{gvt}^k \delta_{gvt}^k \]  

(23)

Dividing (23) by \( \sigma_g - 1 \), thus

\[ \left( \Delta^k \ln p_{gvt} \right)^2 = \frac{\rho_g}{(\sigma_g-1)^2(1-\rho_g)} \left( \Delta^k \ln s_{gvt} \right)^2 + \frac{2\rho_g-1}{(\sigma_g-1)(1-\rho_g)} \left( \Delta^k \ln s_{gvt} \Delta^k \ln p_{gvt} \right) + \frac{\varepsilon_{gvt}^k \delta_{gvt}^k}{\sigma_g-1} \]  

(25)

Thus, we have

\[ Y_{gvt} = \theta_{1g} \times_{1gvt} + \theta_{2g} \times_{2gvt} + u_{gvt} \]

where \( Y_{gvt} = \left( \Delta^k \ln p_{gvt} \right)^2 \), \( X_{1gvt} = \left( \Delta^k \ln s_{gvt} \right)^2 \), \( X_{2gvt} = \left( \Delta^k \ln s_{gvt} \Delta^k \ln p_{gvt} \right) \) and \( u_{gvt} = \frac{\varepsilon_{gvt}^k \delta_{gvt}^k}{\sigma_g-1} \)

and coefficients in terms of \( \rho_g \) and \( \sigma_g \) as follows: \( \theta_{1g} = \frac{\rho_g}{(\sigma_g-1)^2(1-\rho_g)} \) and \( \theta_{2g} = \frac{2\rho_g-1}{(\sigma_g-1)(1-\rho_g)} \)

In computational setup, as estimation yields values of \( \theta_{1g} \) and \( \theta_{2g} \), it is more convenient to calculate for feasible \( \rho_g \) such that \( \theta_{1g} \) and \( \theta_{2g} \) does not violate parameter assumptions on demand and supply elasticities.

Let us re-arrange \( \theta_{2g} \), thus

\[ \sigma_g - 1 = \frac{2\rho_g-1}{\theta_{2g}(1-\rho_g)} \]  

(26)

Substitute (26) into \( \theta_{1g} \), we have

\[ \theta_{1g} = \frac{\rho_g}{\left( \frac{2\rho_g-1}{\theta_{2g}(1-\rho_g)} \right)^2(1-\rho_g)} \]
\[ \theta_{1g} = \frac{\rho_g (1 - \rho_g) \theta_{2g}^2}{(2 \rho_g - 1)^2} \]

\[ \frac{\theta_{2g}^2}{\theta_{1g}} = \frac{(2 \rho_g - 1)^2}{\rho_g (1 - \rho_g)} \]

\[ \frac{\theta_{2g}^2}{\theta_{1g}} = \frac{4 \rho_g^2 - 4 \rho_g + 1}{\rho_g^2 - \rho_g} \]

\[ \frac{\theta_{2g}^2}{\theta_{1g}} = 4 + \frac{1}{\rho_g^2 - \rho_g} \]

\[ \frac{1}{\rho_g^2 - \rho_g} = -\left(4 + \frac{\theta_{2g}^2}{\theta_{1g}}\right) \]

\[ \rho_g^2 - \rho_g = -\frac{1}{\left(4 + \frac{\theta_{2g}^2}{\theta_{1g}}\right)} \]

Plus 0.25 on both sides,
\[ \rho_g^2 - \rho_g + 0.25 = 0.25 - \frac{1}{\left(4 + \frac{\theta_{2g}^2}{\theta_{1g}}\right)} \]

\[ \left(\rho_g - 0.5\right)^2 = 0.25 - \frac{1}{\left(4 + \frac{\theta_{2g}^2}{\theta_{1g}}\right)} \]

Thus,
\[ \rho_g = 0.5 \pm \sqrt{0.25 - \frac{1}{\left(4 + \frac{\theta_{2g}^2}{\theta_{1g}}\right)}} \]