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Average growth rate: Computation methods

This issue of Stats Brief will aim to introduce some of the most common methods to compute average growth rates for time series data, and illustrate the impact of applying different methods for calculating average annual growth rates for GDP per capita and exports of merchandise. Statistical literature introduces several different methods, but there are no solid recommendations on which should be used under which circumstances. However, different methods may result in substantial differences in computed average growth rates.

Growth rates, in general, express changes in values of a variable between two (or more) periods of time. Growth rates are widely published by statistical organizations, and it is popular among media outlets to report on growth or decline of various social or economic phenomena.

To quote growth rates correctly, a set of rules and standards should be respected. For example, growth rates should always be accompanied with explanation of the underlying data, the method used, and the period and horizon used, e.g. quarter-on-quarter, year-on-year, etc. [1] However, much data is published without specification of the method used, or with the use of inconsistent terminology. This leads to different growth rates being quoted for the same time series and same period.

Statistical theory introduces different methods that can be applied to compute growth rates between two or more periods of time. The appropriateness of each method for a given time series depends on the pattern of the differences in value between two successive periods (increments), and whether these increments are constant or changing. The most commonly used reference patterns are [6]:

- *Arithmetic* growth means that the variable of interest changes (increases or decreases) in every period by a constant amount. This represents a linear trend.
- *Geometric* growth occurs when the observed variable changes by a constant ratio from one period to the other. This means that the incremental changes in the variable become larger. Such growth is particularly useful for compounding of monthly, quarterly or annual interest.
- *Exponential* growth assumes that growth compounds continuously at every instant of time, which means that the geometric growth is a special case of exponential growth. Plotting of increments results in a smooth curve because the change is continuous.

It is also worth mentioning that the growth rates are frequently presented in terms of change over the previous time period, e.g. year, quarter, and month. This can be described by different terminology, such as "annual" growth rates (rate of change over previous year, i.e. Y_t/Y_{t-1}), "quarter-on-previous-quarter" (rate of change expressed with respect to the previous quarter, i.e. Q_t/Q_{t-1}), or "month-on-previous-month" (with respect to the previous month, i.e. M_t/M_{t-1}). However, "year-on-year" growth rates are changes expressed over the corresponding period (month or quarter depending on the data) of the previous year, i.e. Q_t/Q_{t-4} or M_t/M_{t-12} [6].

Average growth rate computation methods

Suppose one wants to measure the average growth rate of variable X over n-periods in time, say $X_0, X_1, \ldots, X_n$. Where variable X can be any variable of interest and the n-periods can be defined as any discrete measure of time, such as days, months or years. Statistical literature presents many different methods to compute the average growth rates and here we aim to present some of the most commonly used ones. Conceptually, these methods differentiate themselves based on an assumption on the patterns of evolution of the variable, as described above, and on the weighting structure the method gives to the observations in the time series of interest; that is, the weight each time observation of the variable is given in the computation of the average growth rate.

Table 1 below introduces the four most commonly used methods to calculate average growth rates. The first three methods differentiate themselves on the assumption of the growth patterns, i.e. arithmetic, geometric, or exponential. Also, they take into consideration only the first and the last periods, disregarding the values in between. The fourth method is based on a linear regression trend line fitted to the observations of the time series; hence, it takes into consideration all the intermediate values.

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Table 1: Main average growth rate methods

<table>
<tr><th>Method</th><th>Formula</th><th>Notes</th></tr>
<tr><td>Arithmetic growth rates</td><td>$$r_{AVG} = \left(\frac{X_n}{X_0} - 1\right) / n$$</td><td>Arithmetic growth rate method assumes that the variable of interest increases by a fixed amount of units in each period.

This method takes into account only the first and last observation of the time series, and not the intermediate values.

Arithmetic growth rate is not very widely used, due to the simplistic assumptions. [6]</td></tr>
<tr><td>Geometric growth rates</td><td>$$r_{GEO} = \left(\frac{X_n}{X_0}\right)^{\frac{1}{n}} - 1$$
Which is derived from the compound growth formula (that defines the geometric series):
$$X_n = X_0(1+r)^n$$</td><td>The geometric growth rate represents compound growth over discrete periods, where the changes between two periods differ by a constant ratio. This method is a special case of exponential growth (the compounding periods are longer than infinitesimals and can be of any discreet lengths, e.g. year, month, day, etc.).

This method takes into account only the first and last observation of the time series, and not the intermediate values.

It is also referred to as the geometric average method, as it can be expressed as the geometric average of annual growth rates. Hence, for 1-period interval geometric and arithmetic growth rates are equal, as the arithmetic and geometric formulae become equal.

Geometric growth rate is widely used for indicators on economic phenomena, such as GDP or trade. [4, 5, 6, 7]</td></tr>
<tr><td>Exponential growth rates</td><td>$$r_{EXP} = \ln\left(\frac{X_n}{X_0}\right) / n$$
Which is derived from the general model of exponential growth:
$$X_n = \exp(nr) X_0$$</td><td>Exponential growth rate method represents the limiting case of compounding; that is the compounding takes place continuously (the variable grows at a constant rate at every infinitesimal of time).

This method takes into account only the first and last observation of the time series, and not the intermediate values.

Exponential growth rate will not correspond to the annual growth rate measured at one-year interval by: $(X_n - X_{n-1})/X_{n-1}$, such as arithmetic or geometric rates do.

Exponential growth is mainly used for indicators related to population. [6, 7]</td></tr>
<tr><td>Least-squares growth rates</td><td>$$r_{OLS} = \exp(\hat{\beta}) - 1$$
Which is obtained by estimating parameters of the time trend equation:
$$\ln X_n = \alpha + \beta n + \varepsilon$$
Where,
$$\alpha = \ln X_0$$
$$\beta = \ln(1+\mathrm{r})$$</td><td>The time trend equation is obtained through a logarithmic transformation of the compound growth equation:
$$X_n = X_0(1+r)^n$$
$$\ln X_n = \ln X_0 + n\ln(1+r)$$
This method takes into consideration all values during the time period of interest and gives maximum weight to the growth rates at the middle of the time series.

This growth rate is an average rate and is representative of the available observations over the entire period. It does not necessarily match the actual growth rate between any two periods.

The least-squares growth rate can be used for any type of indicators as it does not assume any pattern of growth. [4, 5, 6, 7]</td></tr>
</table>

Note: n is the number of periods; X_0 is the value of the variable X at time 0; X_n is the value of the variable X at time n; r is the average growth rate over the n-period time series; and ln is the natural logarithm.

It also means that if we apply the average growth rate to the starting value of the time series for the required number of periods (n-periods the growth rate was calculated for), we will not obtain the end value. This is due to the growth rate being the slope coefficient of the trend line with the best fit to the data points.

Use of average growth rates in practice

The prevailing methods seem to be the least-squares and geometric growth rates. Kakwani [4] states that the least-squared method is the most commonly used average growth rate method. However, sometimes it is reasonable to differentiate the use of growth rate methods based on the underlying variable in question. For example, for economic variables the most common method used is the geometric growth rate, as economic variables are measured at intervals; whereas for human population change the exponential growth rates are mainly used.

Organizations in their publications use different methods for presenting average growth rates. Unfortunately, in many publications the method used is not made clear. Below we gather examples of use of different growth rate methods in publications of some international statistical organizations:

- ESCAP: geometric growth rates are used for all indicators in all statistical publications and the online statistical database.
- The World Bank: least-squares growth rates are used if there is a sufficiently long time series, otherwise the exponential or geometric growth rates are used (World Bank, 2015a). For all international trade time series, the geometric growth rates are used (World Bank, 2015b).
- FAO: geometric growth rates are used for shorter time series and least-square growth rates for longer time series (FAO, 2013).
- IMF: geometric growth rates are used for most time series, except for unemployment the arithmetic growth rates are used (IMF, 2015).
- United Nations Statistics Division: geometric growth rates are used for GDP time series (UNSD, 2015).
- United Nations Population Division: exponential growth rates are used for population time series (UNPD, 2013).

Data example and comparison of methods

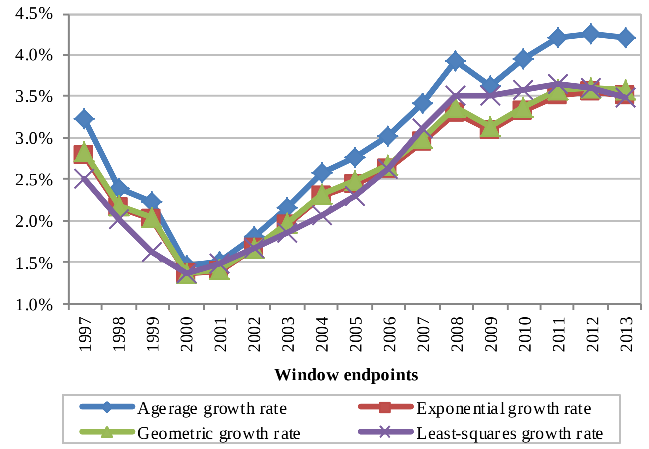
For illustration purposes, average growth rates for 10-year moving window periods have been compared for two time series. The first time series is the per capita gross domestic product (GDP) in constant prices (2005 US dollars) for all economies in Asia-Pacific between 1987 and 2013. Table 2 presents the average growth rates calculated using the four methods outlined above in Table 1. The results are also illustrated in Figure 1, which plots the window endpoints for the 10-year moving window periods on the horizontal axis.

Table 2: Average annual growth rates over 10-year windows, GDP per capita

	Average annual growth rate (% per annum)			
10-year window	Arithmetic growth rate	Exponential growth rate	Geometric growth rate	Least-squares growth rate
1987-1997	3.2	2.8	2.8	2.5
1988-1998	2.4	2.2	2.2	2.0
1989-1999	2.2	2.0	2.0	1.6
1990-2000	1.5	1.4	1.4	1.4
1991-2001	1.5	1.4	1.4	1.5
1992-2002	1.8	1.7	1.7	1.7
1993-2003	2.2	2.0	2.0	1.9
1994-2004	2.6	2.3	2.3	2.1
1995-2005	2.8	2.4	2.5	2.3
1996-2006	3.0	2.6	2.7	2.6
1997-2007	3.4	2.9	3.0	3.1
1998-2008	3.9	3.3	3.4	3.5
1999-2009	3.6	3.1	3.1	3.5
2000-2010	3.9	3.3	3.4	3.6
2001-2011	4.2	3.5	3.6	3.7
2002-2012	4.3	3.5	3.6	3.6
2003-2013	4.2	3.5	3.6	3.5

Source: ESCAP Statistical Database.

Figure 1: Average annual growth rates over 10-year windows, GDP per capita (% per annum)



Source: ESCAP calculations.

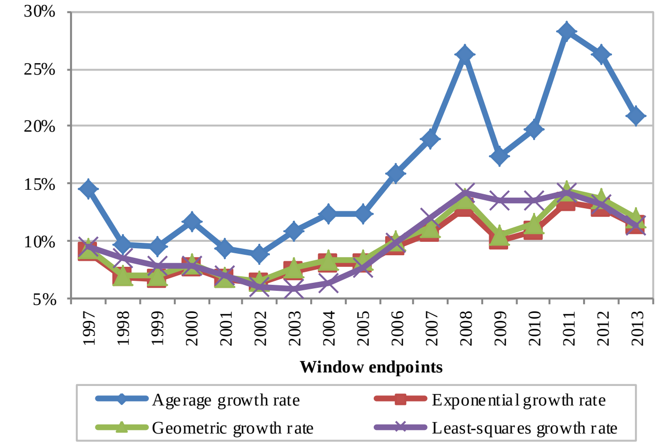
The second time series analysed is the annual values of total exports of merchandise from all economies in Asia-Pacific in US dollar values between 1987 and 2013. The average growth rates using the four methods described above are presented in Table 3 and illustrated in Figure 2, which plots the window endpoints for the 10-year moving window periods on the horizontal axis.

Table 3: Average annual growth rates over 10-year windows, exports of merchandise

	Average annual growth rate (% per annum)			
10-year window	Arithmetic	Exponential growth rate	Geometric growth rate	Least-squares growth rate
1987-1997	14.6	9.0	9.4	9.6
1988-1998	9.6	6.8	7.0	8.5
1989-1999	9.5	6.7	6.9	7.9
1990-2000	11.6	7.7	8.0	7.8
1991-2001	9.4	6.6	6.9	7.0
1992-2002	8.9	6.4	6.6	6.0
1993-2003	10.9	7.4	7.7	5.8
1994-2004	12.4	8.0	8.4	6.3
1995-2005	12.4	8.0	8.4	7.7
1996-2006	16.0	9.5	10.0	9.9
1997-2007	18.9	10.6	11.2	12.1
1998-2008	26.3	12.9	13.8	14.2
1999-2009	17.3	10.1	10.6	13.6
2000-2010	19.7	10.9	11.5	13.5
2001-2011	28.3	13.4	14.4	14.2
2002-2012	26.3	12.9	13.8	13.2
2003-2013	20.9	11.3	11.9	11.4

Source: ESCAP Statistical Database.

Figure 2: Average annual growth rates over 10-year windows, exports of merchandise (% per annum)



Source: ESCAP calculations.

In both cases, we can see substantial differences between the growth rates presented. In general, both exponential and geometric growth rates are very closely following each other. We can also observe that all three methods that take only the first and last value into consideration (arithmetic, geometric, exponential) are more reactive to temporary changes in the time series; the arithmetic growth rate in particular. This is especially visible in the second example with the exports of merchandise data, where there was a significant decrease in 2009. Whereas, on the other hand the least-squares growth rates feature a much smoother curve and the sudden effects in the time series do not impact the average growth rates in the same way. In both of these two examples geometric, exponential or least-square methods all adequately reflect the movement of the underlying variables. Since both of the economic time series above are measured at discrete intervals, hence the geometric growth model may be most appropriate.

In conclusion, we can say that the selection of average growth method depends mainly on the underlying variable in question and the availability of data. The actual choice is not strongly prescribed by the literature, and is, as such, left to the discretion of the producer. More important, however, is that the producers of statistics clearly state which method was used, and allow the reader to understand the underlying assumptions and avoid confusions.

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